NEW ECONOMIC WINDOWS

Marisa Faggini
Concetto Paolo Vinci (Eds.)

Decision Theory and Choices: a Complexity Approach

Springer

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New Economic Windows

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Editors

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Preface

In Economics most empirical models of decision-making assume that the economic agents act with rationality when making choices and that their choices are aimed at optimizing their utility or profit. What is taken as "rational" is of chief importance because rationality is used either to decide which course of action would be the best to take, or to predict which course of action actually will be taken.

In a more formal sense the economic agents are said to have transitive and consistent preferences and seek to maximize the utility that they derive from those preferences, subject to various constraints. They operate according to the choice imperative: given a set of alternatives, choose the best.

This process of choice postulates utility values associated with possible states of the world perfectly foreseen in which situations with higher utilities are preferred to those with lower ones. Those preferences are defined over outcomes, known and fixed, so that decision makers maximize their net benefits by ordering and choosing the alternative that yields the highest level of benefits. Choices among competing goals are handled by indifference curves – generally postulated to be smooth (twice differentiable) – that specify substitutability among goals.

But, more important, this long and complicated decision process is realized with perfect foresight in the time agent acts. Agent rationality and homogeneity, non-interactiveness and equilibrium are some of the basic assumptions of the so-called Rational Choice Theory.

The Rational Choice Theory plays a central role in Economic Theory. It just considers that "explanations are regarded as economic to the extent that they explain the relevant phenomena in terms of the rational choice of individual economic agents" (Sugden, 1991). Any other theory which doesn't have the same structure is not considered.

But is the Theory of Rational Choice adequate to describe a world in which agents use inductive rules of thumb to make decisions, have incomplete information, are subject to errors and biases, learn to adapt over time, are heterogeneous, interact with one another, in a few words are not rational in a conventional sense? That is, are the basic tenets of Rational Choice Theory a good description of economic agents' real behavior?

These questions arise because in this theory some rather unrealistic assumptions are made about the world: an individual has precise information about what will occur under any choice made; an individual has time and ability to weigh every choice against every other choice and finally an individual is fully aware of all possible choices. Further, individual preferences are taken to be given a priori, rather than constructed and revised through ongoing social processes: they are primitive, consistent, and immutable.

In most rational-choice models social phenomena are decomposed into sequences of individual actions set aside culture, psychology, class, group dynamics, or other variables that suggest the heterogeneity of human behavior. Economic actors are treated as equivalent or the differences among them are presented as attributes of individuals. The regularities not the differences are considered. Of course, these could be incorporated into analyses, but much of the power and elegance of rational-choice models depends on these simplifying assumptions.

The hypothesis are totally unrealistic because they don't reflect real individual behavior (Robles, 2007), nor does it reflect the complexity of human decision making (Shapiro and Green, 1994).

It is clear that the foundations of Rational Choice Theory are less secure than we thought, so they need to be re-examined and perhaps rebuilt (Sugden, 1991).

Many researchers have put forward the idea that consumer choice behavior can be affected by environment complexity. Human behavior in this context can show many effects which are contrary to predictions of the standard economic framework. Such effects include those arising from social preferences (how people deal with each other), individual bounded rationality (how people make mistakes in decisions) and uncertainty (Kahneman & Tversky, 1979).

Consequently economists have been forced to choose between an elegant but incorrect theory and an inelegant collection of empirical observations.

In 1944, von Neumann and Morgenstern made a first step in this direction; they provided a variant of rational choice formalizing the Expected Utility Hypothesis. They integrated risk and uncertainty into the model by associating a probability distribution, estimated by the decision maker but again the agent maximizes not more utility but expected utility. Further the individual actors are conceptualized as decision makers whose choice depend directly on the decisions of others.

This conceptualization is quite different from the mainstream of Rational Choice models in which individuals only interact via market clearing prices. Nevertheless in this game (interactions between individuals) the players, affected by well-defined (and not changing) preferences over outcomes of decision, choose strategies that will maximize their individual expected utilities.

Later on the decision making process was seriously questioned from another viewpoint and context: that of the ability to make rational calculations. We are moving now from economic models of decision making in which an anonymous rational agent performs omniscient probability calculations with unlimited cognitive resources to models with agents who have limited processing capabilities. In more formal words their rationality is bounded (Simon, 1982, 1997).

The focal point of Bounded Rationality Theory is not that economic agents might decide differently if they had more or different information, or differently with different items in the utility function. Rather it is that they can't process all the information even if they had it. In this case the agents choose optimally, from their set of strategic alternatives, taking into account only a subset of the set of possible future contingencies.

These cognitive limitations can lead people to make suboptimal choices (Newell and Simon, 1972). Such choices can arise for a variety of reasons, including simply not understanding or being aware of the options, mistakes in evaluating the outcomes, and various biases based on how the choices are described (Kahneman and Tversky, 1979).

But this further step is not yet enough to analyze the behavior of economic agents in a world complicated not only by actions of other ever changing agents but also by the actions of environment on them. Decision-models based on bounded rationality are still inefficient under conditions of Complexity Theory because rather than an alternative concept it is a weaker formulation of the rationality postulate.

The view of Complexity Theory focuses on uncertainty, limited cognitive, heterogeneity and processes. Decision makers have to face diversity, unpredictability, self-reference, change and Knightian uncertainty (Fontana, 2008). While it could be the case that the assumption of rational behavior is credible for a small subset of people, it is certainly the case that not all agents are equally rational, as is implicit in conventional theoretical models.

The assumptions of mainstream economics are totally changing. No longer the Olympic rationality but processes in which the interacting economic agents adapt themselves in reaction to environment and, by innovating, contribute to its change. The aim is to improve the performance of actions updating the information so that new options are generated.

In this ever changing environment, it is almost impossible to prefigure the outcome of decisions to a satisfactory degree of precision and use constrained optimization models to capture the behavior of these complex adaptive systems.

In mainstream of traditional economic theory agents interact only by impersonal markets and not with one another, so that the interaction structures are very simple. From the complexity point of view all economic actions involve interactions among agents. Not more aggregate reduced to the analysis of a single, representative, individual, ignoring by construction any form of heterogeneity and interaction, but the aggregate emerging from the local interactions of agents.

In this book there are no general surveys of the decision making problem, rather each contribution discusses a very specific aspect of how complexity affects human decisions and which tools have to be used to model the decision-making process in a complex scenario. The thirteen chapters reflect this end from various modeling perspectives such as agent-based models, fuzzy theory, neural networks, societal networks, and thermodynamic cycles.

They are divided in five sections: General Issue, Agent based model, Techniques and Tools, Modeling from Physics and Related Issues.

In the General Issues, the first three papers by Arecchi, Velupillai and Bruno emphasize that the agents don't choose in the standard sense. Arecchi describes the processing and the cognition of visual events when complexity, in terms of deterministic chaos, occurs. Velupillai reformulates bounded rationality and satisficing in a computable framework so that their intrinsic complex dynamics are made explicit in as straightforward a way as possible. Bruno highlights how the optimization in the standard sense is unsuitable for complex choices, where the identification of measurable objectives and their relative weights cannot be objective and stable through time.

In the next set of papers by Terna, Biggiero and Chiarella et al., the application of Agent Based Models are proposed. Terna proposes a simplified application of the original Swarm Protocol denominated Swarm Like Agent Protocol in Python to the actual important issue of interbank payment and liquidity. Biggiero uses the agent-based simulation model for analyzing the effects on industry profitability of the two exploration modes for choosing the best suppliers. Starting from the consideration that the complexity of economic cycles is the result of the impact of individual decisions on macro variables of the economy Chiarella et alt. provide micro-economic foundations to the model of the financial fragility designed as an heterogeneous agent model.

Part III is focused on Techniques and Tools from the Complexity Theory. The Lafuente and Di Tollo and Lyra papers describe how decision process characterized by limited information, limited in situations of uncertainty computational skills, adaptation and induction can be modeled using tools like fuzzy systems and neural nets. In Lafuente's paper the uncertainty in which the financial operators move could be manage using fuzzy systems. Di Tollo and Lyra show that Neural Nets can be very successful in order to forecast the credit risk assessment.

Part IV is about Modeling from Physics to explain decision process. The acquisition of information by the methods of chemical kinetics are described in Monaco's contribution while in Khrennikov the financial markets dynamics are discussed from the point of view of phenomenological thermodynamics. In Meucci et al. how decision maker could control a complex system is shown.

The final three papers are about related issues to Complexity Theory springing from the networks pervasive in the study of complex systems to the building inequality indices. Abatemarco investigates the relationships between deprivation and complaint-based inequality orderings highlighting that the intensity of inequality is defined as an aggregation of individual perceptions of inequality. In the paper by Bimonte new models and techniques that deal with social networks characterized by self-organized criticality are presented. The risk assessment based on the definition of risk ontologies is investigated by Nota et alt. in the network of interacting systems.

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Part I
General Issues

Coherence, Complexity and Creativity: the Dynamics of Decision Making

Fortunato Tito Arecchi

Abstract. ***Coherence*** is a long range order (either in space or time) which stimulates our curiosity and drives the scientific investigation. To build non-trivial correlations, a system must be away from thermal equilibrium; this implies entering a nonlinear dynamical regime, where coherence is just one aspect. The coupling of many partners leads to a multiplicity of equilibrium states, the number of which increases exponentially with the number of partners; we call ***complexity*** such a situation. Complete exploration of *complexity* would require a very large amount of time. On the contrary, in cognitive tasks, one reaches a decision within a short time. Indeed, any conscious perception requires a few hundred milliseconds. It is characterized by a collective neuron *synchronization*. However, the loss of information in the chaotic spike train of a single neuron takes a few msec; thus perception implies a *control of chaos*, whereby the information survives for a time sufficient to elicit a decision.

Control of chaos is achieved by the combination of a *bottom-up* sensorial stimulus with a *top-down* perturbation induced by the semantic memory. *Control of chaos* has an optimality: indeed if too short no decisions emerge, if too long it blocks the availability to sequential cognitive tasks. We call ***creativity*** this optimal control . We extrapolate the same behaviour from perceptual tasks (sensorial input) to cognitive tasks (conceptual input), going beyond the Bayesian inference, which is the way a computer operates.

Control of chaos in a cognitive dynamics overtakes the limitation of Newton-Laplace determinism, since the cognitive agent re-codes its dynamical space . Such a re-coding is a very individual free endeavour; it is the basis of *creativity*.

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1 Introduction

As already delineated in the Summary, the three terms in the title are defined as follows:

Coherence = long range order (in space [vision] or time [music]);
Complexity = display of different coherences;
Creativity = jump from one coherence regime to another.

What are the relations among the three concepts? A detailed analysis has been given elsewhere (Arecchi, 2007) and here we recall the main points. *Coherence* is associated with long range correlations, in space or time; *complexity* arises whenever an array of coupled dynamical systems displays multiple paths of coherence. *Creativity* corresponds to a selection of a coherence path within a complex nest. As we will see soon, it seems dynamically related to *control of chaos*.

Exploration of a *complex* situation would require a very large amount of time, in order to classify all possible *coherences*, i.e. long range correlations. In cognitive tasks facing a *complex* scenario, our strategy consists in attaining a decision within a finite short time. Any conscious perception (we define conscious as that eliciting a decision) requires a few hundred milliseconds, whereas the loss of information in the chaotic spike train of a single neuron takes a few msec.

The interaction of a *bottom-up* signal (external stimulus) with a *top-down* modification of the control parameters (induced by the semantic memory) leads to a collective synchronization lasting a sizable fraction of a second: this is the indicator of a conscious perception. The operation is a *control of chaos,* and it has an optimality; if it lasts less than 200 msec, no decisions emerge, if it lasts much longer, there is no room for sequential cognitive tasks. We call *creativity* this optimal control of neuronal chaos. It amounts to selecting one among a large number of possible coherences all present in a complex situation. ***The selected coherence is the meaning of the object under study***.

In Section 2 we survey the main steps of a perceptual process, with reference to vision. In Section 3 we discuss the cognitive process, starting from the most fundamental inference based on Bayes theorem; we show that it is limited to a single model (or likelihood) whereas complex situations require changes of model. This is the creative step peculiar of human decisions and impossible to a Turing machine. In Section 4 we compare two pairs of conflicting categories relevant for decision making, namely, *partial truth* versus *global truth* and *hermeneutic circle* versus *hermeneutic spiral*.

2 How We See: Processing Visual Events

Figure 1 shows a brain section with the path of the visual information. A given light signal impinging on the eye's retina stimulates the production of electrical pulses (spikes) high about 100 millivolt and lasting about 1 millisecond. The time sequence

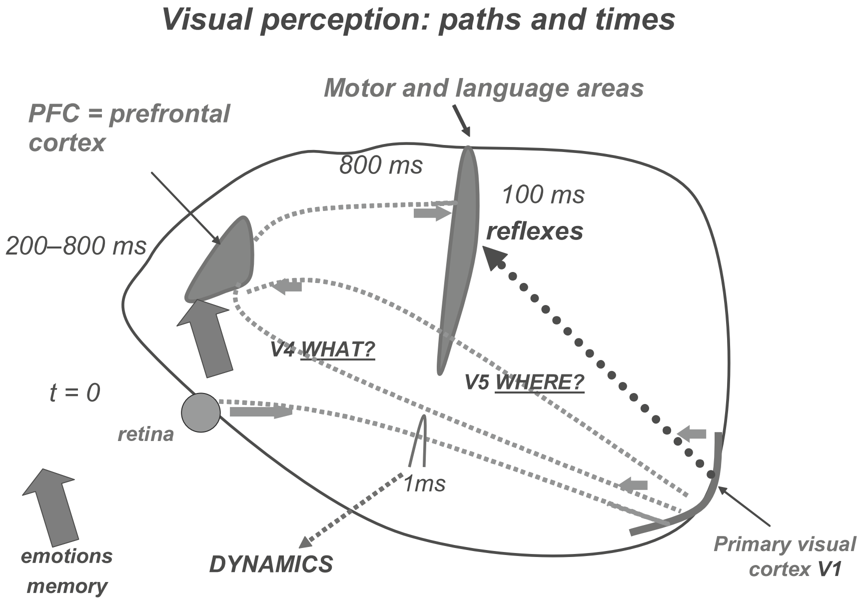


Fig. 1 Time sequence of brain events following a visual stimulus on the retina at time $t = 0$. After 200 msec, the signal coded as a train of electrical pulses of 1 msec duration (spikes) traveling on neurons' axons, arrives at the primary visual cortex V1; the dynamics of spikes (interspike interval) depends on the sensory stimuli and on the receptive cell (Fig. 2). The signal from V1 is then elaborated in two distinct areas, V4 or **WHAT** area which recognizes shapes and color, and V5 or **WHERE** area, which recognizes motion and space relations. The separate information is combined in the PFC (prefrontal cortex) together with top-down signals coming from the inner brain (emotions, memory). This mixing takes about half a second starting from 200 msec; at 800 msec after the sensitization of the retina, a decision emerges activating the motor and language areas. The elaboration in PFC takes so long because the input is compared with previous memories and hence re-coded. Instead vital responses arrive directly to the motor area in even less than 100 msec (**reflexes**: *heavy dotted line*)

of these standard pulses represents a neural code, this means that different inputs are coded in different spike sequences. Spikes travel on the neuron axons, like on transmission lines At the axon end (synapse) the electrical signal releases some amount of neurotransmitters, that is, specialized chemicals which diffuse in the inter-neuron space and arrive to a next neuron, where they again stimulate electrical spikes and so on. The brain has a huge number of neurons tangled by a web of mutual couplings. A detailed account of all couplings is out of reach at the present state of neuroscientific investigation. It would require resolving each neuron separately; this can be done in laboratory animals by inserting a sparse number of microelectrodes each one probing a single neuron. Coarse grained information is captured by EEG (electro encephalo-gram) whereby we measure the electrical activity probed by some electrodes glued on the scalp. At variance with the microelectrodes probing the single neuron, the technique is non invasive; however an external electrode sums up

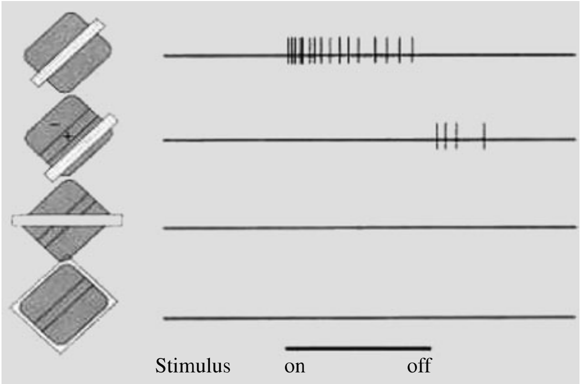


Fig. 2 Sensitive cell detecting light. If light impinges on the + area, the number of output spikes (*vertical bars* plotted versus time) gets large. If light impinges on the − area or is spread over both areas, then the no output emerges

the activity of many neurons, with a poor space resolution. The fMRI (functional magnetic resonance imaging) is sensitive to the magnetic properties of oxygenated blood; thus it records the blood intake to a brain area which is in operation and hence requires energy; again, the space resolution is poor.

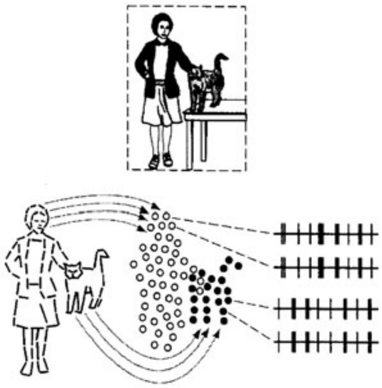
Let us consider the visual system; the role of elementary feature detectors has been extensively studied (Hubel, 1995). In Fig. 2 we show how various stimulus geometries evoke different responses in a cell with receptive field $+/-$. The bright slit is turned on and, 1 second later, turned off.

Top record: response to a slit of optimum size, position, and orientation. Second line, the slit covers only part of an inhibitory area. Third, the slit is oriented so as to cover only a small part of the excitatory region. Fourth, the whole receptive field is illuminated; again, no response.

Thus, through experiments like these we know that some neurons are specialized in detecting exclusively vertical or horizontal or tilted bars, or a specific luminance contrast, etc.

However the problem arises: how elementary detectors contribute to a holistic (*Gestalt*) perception? A hint is provided by (Singer et al., 1995). Suppose we are exposed to a visual field containing two separate objects. Both objects are made of the same visual elements, horizontal and vertical contour bars, different degrees of luminance, etc. What are then the neural correlates of the identification of the two objects? We have one million fibers connecting the retina to the visual cortex. Each fiber results from the merging of approximately 100 retinal detectors (rods and cones) and as a result it has its own receptive field. Each receptive field isolates a specific detail of an object. We thus split an image into a mosaic of adjacent receptive fields. Now the "*feature binding*" hypothesis consists of assuming that all the cortical neurons whose receptive fields are pointing to a specific object synchronize the corresponding spikes, and as a consequence the visual cortex organizes into

Feature binding (W. Singer)



Each circle represents a respective field which detects an elementary detail (e.g. a vertical bar)

Fig. 3 Feature binding: the lady and the cat are respectively represented by the mosaic of *empty and filled circles*, each one representing the receptive field of a neuron group in the visual cortex. Within each *circle* the processing refers to a specific detail (e.g. contour orientation). The relations between details are coded by the temporal correlation among neurons, as shown by the same sequences of electrical pulses for two *filled circles* or two *empty circles*. Neurons referring to the same individual (e.g. the cat) have synchronous discharges, whereas their spikes are uncorrelated with those referring to another individual (the lady) from (Singer et al., 1995)

separate neuron groups oscillating on two distinct spike trains for the two objects. Direct experimental evidence of this synchronization is obtained by insertion of microelectrodes in the cortical tissue of animals just sensing the single neuron (Fig. 3).

The interaction of a *bottom-up* signal (external stimulus) with a *top-down* change of the control parameters (induced by the semantic memory) leads to a collective synchronization lasting 200 msec: this is the indicator of a conscious perception. The operation is a *control of chaos,* and it has an optimality; if it lasts less than 200 msec, no decisions emerge, on the contrary, if it lasts much longer, there is no room for sequential cognitive tasks.

The addition of extra degrees of freedom implies a change of code, thus it can be seen as a new level of description of the same physical system.

Based on the neurodynamical facts reported above, we can understand how this occurs (Grossberg, 1995). The higher cortical stages from where decision emerge have two inputs. One (bottom-up) comes from the sensory detectors via the early stages which classify elementary features. This single input is insufficient, because it would provide the same signal for e.g. horizontal bars belonging indifferently to either one of the two objects. However, as we said already, each neuron is a nonlinear system passing close to a saddle point, and the application of a suitable perturbation can stretch or shrink the interval of time spent around the saddle, and thus lengthen or shorten the interspike interval. The perturbation consists of top-down

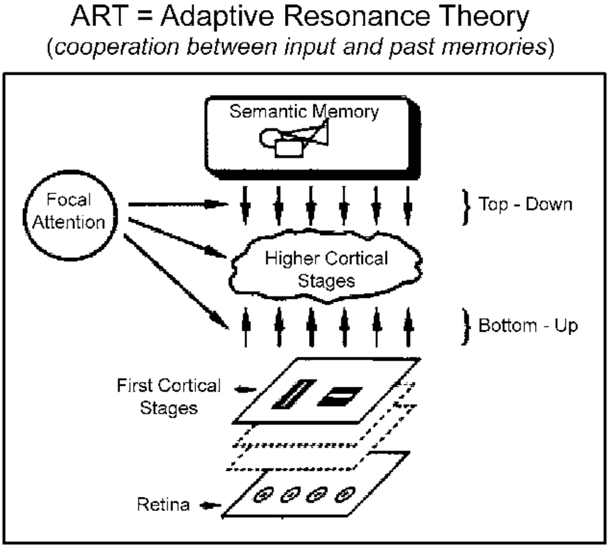


Fig. 4 ART = Adaptive Resonance Theory. Role of bottom-up stimuli from the early visual stages an top-down signals due to expectations formulated by the semantic memory. The focal attention assures the matching (resonance) between the two streams

signals corresponding to conjectures made by the semantic memory (Fig. 4). ***ART (adaptive resonance theory)*** refers to a re-coding of bottom up stimuli based on previous learning.

For simple stimuli which require a fast reaction, there is a direct access from V1 to the motor areas as shown in Fig. 1.

On the other hand, exploration of a *complex* situation would require a very large amount of time. In cognitive tasks facing a *complex* scenario, the cognitive strategy consists in converging to a decision within a finite time. Various experiments (Libet, 1979; Rodriguez et al., 1999) prove that a decision is taken after a few hundred milliseconds of exposure to a sensory stimulus Thus, any conscious perception (we define conscious as that eliciting a decision) requires a few hundred msec, whereas the loss of information in a chaotic train of neural spikes takes a few msec.

There must be a mechanism of chaos control which holds a given information for the time necessary to decide about.

Synchronization of a chain of chaotic lasers provides a promising model for a *physics of cognition.* Indeed, a peculiar type of chaos, consisting of sequences of identical spikes erratically occurring in time, has been discovered in lasers and called HC (Arecchi and Meucci, 2008). The chaotic train can be regularized by synchronization actions. An array of weakly coupled HC systems represents the simplest model for a physical realization of feature binding. The array can achieve a collective synchronized state lasting for a finite time (corresponding to the physiological 200 ms!) if there is a sparse (non global) coupling, if the input (bottom-up) is applied to just a few neurons and if the inter-neuron coupling is suitably adjusted (top-down control of chaos) (Ciszak et al., 2008, 2009).

This approach is discussed in the next section.

3 The Scientific Insight

As Galileo was saying in a letter to Marc Welser, in 1610, a sound scientific approach means avoiding philosophical considerations on the nature of objects, and limiting oneself to measurable data coded in numbers, then connecting these numbers by a mathematical formalism The scientific method of Galileo is based on two pillars:

- **Sensate esperienze = experiments performed through our senses**: if the case helped by equipment, e.g. the telescope and the clock;
- **Necessarie dimostrazioni = compelling proofs**: in a theorem, if one agrees on the premises and understands the proof, than he must necessarily accept the conclusions.

Thus, in science meaning appears irrelevant and only measurable details and repeatable phenomena make sense.

The Galileian innovation was to replace words with numbers. Some time later, Newton discovered that assignment of position and velocity of a particle at a given time (the initial condition) determines univocally its future trajectory. All objects in nature are made of elementary particles endowed of this Newtonian property. Thus the future of the whole object can be predicted once we know its initial condition. This was the basis of Laplace determinism (1812) disproved a few decades later by Poincaré.

3.1 Deterministic Chaos and Its Control

For a Newtonian particle, once we know the forces, the trajectory emerging from a given initial condition, as the star in Fig. 5, is unique. The coordinates of the initial point are in general real numbers truncated to a finite number of digits, thus the initial condition is spread over a small patch. Points of the patch converge toward the calculated trajectory or diverge from it, depending on whether the transverse stability yields a landscape like a valley (left) or the ridge of a hill (right). Poincaré proved that from 3 coupled dynamical degrees of freedom on, the right situation is generic. Nowadays we call it *deterministic chaos*. It is a sensitive dependence on initial conditions and implies a loss of information of the initial preparation. The loss rate K is called Kolmogorov entropy. We can adjust its value by adding extra variables which change the slope of the downhill fall without practically perturbing the longitudinal trajectory (control of chaos), as shown in Fig. 6. Control of chaos

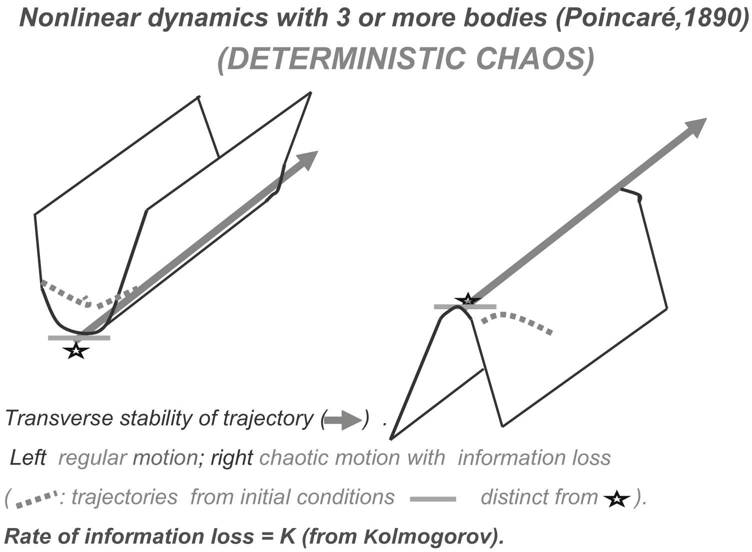


Fig. 5 Deterministic chaos-the trajectory emerging from a precise initial condition, as the star, is unique. However, in general the initial condition is spread over a small patch. Points of the patch converge toward the calculated trajectory or diverge from it, depending on whether the transverse stability landscape is a valley (*left*) or a hill (*right*). From 3 coupled dynamical degrees of freedom on, the right situation is generic. we call it deterministic chaos. The information loss rate K is called Kolmogorov entropy

occurs everywhere in interacting systems and it is essential to establish coherent features.

Suppose we have a N-dimensional chaotic system: adding a few, say p, extra variables we embed the dynamical problem in a $(N + p)$-dimensional space. By careful choice of the control, we can keep the longitudinal trajectory of the original dynamics, modifying however its transverse stability. In the example of Fig. 6, we consider a perceptual feature encoded by a chaotic dynamics which loses information over 2 msec. This time is insufficient to elicit a decision; thus a mechanism of chaos control must stabilize chaos for about 200 msec. The chaos control was initially devised for complete stabilization (as shown on the left of Fig. 6). this strategy however is unfit for perceptual purposes, since the agent should be able to react to a sequence of stimuli, thus it has to fix a given input just for the time necessary to decide about, and then be ready for the next input.

We thus hypothesize that ***transient control of chaos*** is the strategy whereby a cognitive agent exploits previously learned resources in order to re-code the input arriving from the early sensory stages. Re-coding over a transient interval can be done in multiple ways. The selected way is the one that best fits the interaction of the agent with its environment. We elaborate this hypothesis in the following sub-sections.

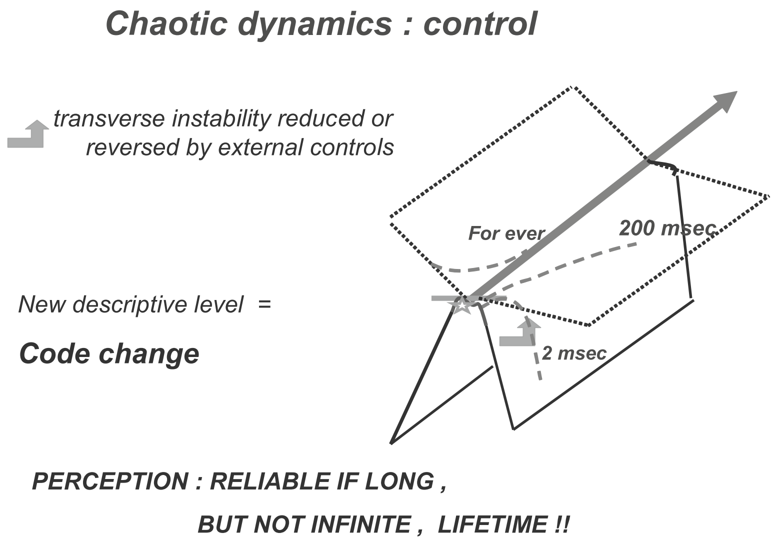


Fig. 6 Chaos is controlled by adding extra-dynamic variables, which change the transverse instability without affecting the longitudinal trajectory. In the perceptual case, the most suitable top-down signals are those which provide a synchronized neuron array with an information lifetime sufficient to activate successive decisional areas (e.g. 200 ms), whereas the single HC neuron has a chaotic lifetime of 2 ms. If our attentional-emotional system is excessively cautious, it provides a top-down correction which may stabilize the transverse instability for ever, but then the perceptual area is blocked to further perceptions

3.2 Stabilizing a Perception (Sensorial Input)

From a dynamical point of view, the single neuron is a chaotic system. Even though the electrical spikes are all equal to each other, their time separation is chaotic. As a general consideration, a living being is a semiotic agent; by this we mean that the agent, embedded in an environment, changes the descriptive code in order to stabilize the cognitive dynamics and compensate for unavoidable information loss associated with the chaotic dynamics of its neurons. We have already stated that any conscious perception (we define conscious as that eliciting a decision) requires about 200 msec, whereas the loss of information in a chaotic train of neural spikes takes a few msec.

We have already seen Singer experiment (Fig. 3) on feature binding.

An array of weakly coupled HC systems represents the simplest model for a physical realization of feature binding. The array can achieve a collective synchronized state lasting for a finite time (corresponding to the physiological 200 ms!) if there is a sparse (non global) coupling, if the input (bottom-up) is applied to just a few neurons and if the inter-neuron coupling is suitably adjusted (top-down control of chaos) (Arecchi, 2004; Ciszak et al., 2009).

The operation is a *control of chaos*, and it has an optimality; if it lasts less than 200 msec, no decisions emerge, on the contrary, if it lasts much longer, there is no room for sequential cognitive tasks.

The addition of extra degrees of freedom implies a change of code, thus it can be seen as a new level of description of the same input.

3.3 From Perception to Cognition – Creativity

Considerations analogous to those developed for perception (elaboration of a sensorial input) can be made for cognition (conceptual input).

We distinguish two types of cognitive task. In *type I*, we work within a prefixed framework and readjust the hypotheses at each new cognitive session, by a Bayes strategy. Bayes theorem [Bayes] consists of the relation:

$$\boldsymbol{P(h|data) = P(h)[P(data|h)/P(data)]} \tag{1}$$

That is: the probability $P(h|data)$ of an hypothesis h, conditioned by the observed *data* (this is the meaning of the bar |), called *a-posteriori probability of h*, is the product of the a-priori probability $P(h)$ of that hypothesis (we assume to have guessed a package of convenient hypotheses with different probabilities), times the ratio of the probability $P(data|h)$ that *data* is generated by an hypothesis h, (this is *the model*) to the probabilty $P(data)$ of the effectively occurred data. When this ratio is largest, the a-posteriori probability is maximum. As shown in Fig. 7, starting from an initial observation and formulating a large number of different hypotheses, the one supported by the experiment suggests the most appropriate dynamical explanation. Going a step forward and repeating the Bayes procedure amounts to climbing a probability mountain along a steepest gradient line.

Recent neurological studies (Beck, 2008) explain the readiness of reflexes in terms of a fast coding done by a Bayesian strategy which operates very fast, in about 100 msec.

Thus fast reactions are NOT the result of an interplay with previous semantic memories at the PCF level (Fig. 1) as modeled in ART (Fig. 4).

Fast indeed,but: how accurate? What confidence is provided by a Bayesian procedure?

To answer the question, we must consider a new feature, namely ***complexity***.

A complex problem is characterized by a probability landscape with many peaks, insofar as it can not be described by a single model (Fig. 8). Jumping from a probability hill to another is non-Bayesian; let us call it *type II* cognition.

In human cognition, *type II* is driven by hints suggested by the context (*semiosis*) yet not included in the model. *Type II* task is a *creativity* act because it implies a change of code, at variance with *type I*, which operates within a fixed code. The ascent to a single peak can be automatized in a steepest gradient program; once the peak has been reached, the program stops, any further step would be a downfall.

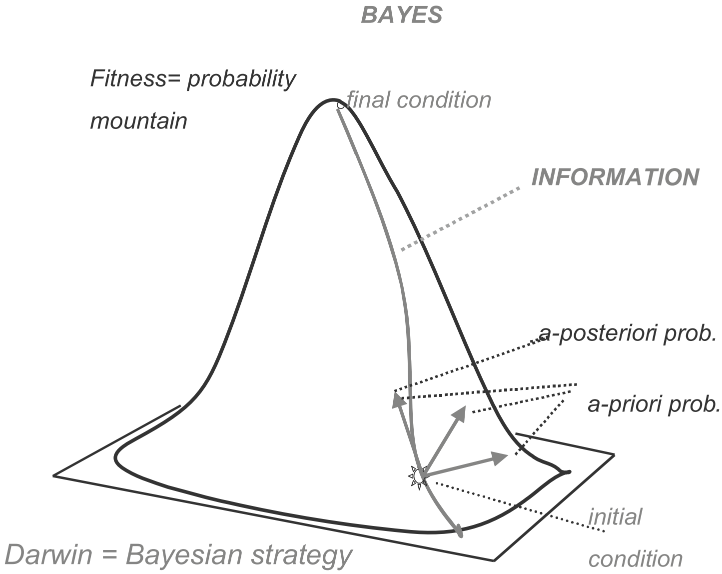


Fig. 7 Successive applications of the Bayes theorem to the experiments. The procedure is an ascent of the Probability Mountain through a steepest gradient line. Each *point* of the *line* carries an information related to the local probability by Shannon formula. Notice that Darwinian evolution by mutation and successive selection of the best fit mutant is a sequential implementation of Bayes theorem

Question: can the changes of hill in Fig. 8 be handled by a computer? Answer: No; indeed, as the computer behavior is perturbed by some added noise (as done in Montecarlo simulations or in genetic algorithms) it explores the immediate surrounding of a Bayes uphill path. A finite step away from it would require a guideline, otherwise it would be whimsical. Thus, a non-deterministic computer can not perform the jumps of *type II*, since it intrinsically lacks semiotic abilities. In order to do that, the computer must be assisted by a human operator. We call "*meaning*" the multi-peak landscape and "*semantic complexity*" the number of peaks.

This intrinsic limitation of a formalism was stated in two famous theorems, namely, the first incompleteness theorem for a consistent theory (Goedel, 1931) and its computer version (Turing, halting problem, 1936) (Fig. 9). Goedel theorem, stating that there are propositions which appear true to the intuition (semiotic appreciation) of a mathematician endowed with the body of axioms, yet they can not be deduced via the syntax of the formalism, signed a sharp end to the formalistic dream of David Hilbert (1900), to explain all mathematics in terms of logic. Since a universal computer is a syntactical machine which grinds the input data via its inner instructions, a similar limitation applies to a computational procedure (Turing). In the space in which we represent Bayesian procedures (Fig. 9), the Goedel–Turing theorems are visualized as true propositions (peaks of left-ward hills), accessible from the axioms by a creative jump but not by via the formalism.

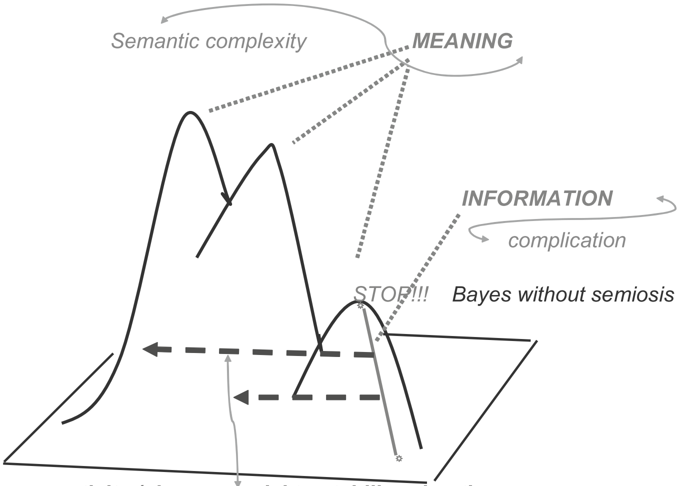


Fig. 8 Semantic complexity – A complex system is one with a many-peak probability landscape. The ascent to a single peak can be automatized by a steepest gradient program. On the contrary, to jump to other peaks, and thus continue the Bayes strategy elsewhere, is a creativity act, implying a holistic comprehension of the surrounding world (semiosis). We call "**meaning**" the multi-peak landscape and "**semantic complexity**" the number of peaks. It has been guessed that semiosis is the property that discriminates living beings from Turing machines [Sebeok]; here we show that a non-algorithmic procedure, that is, a non-Bayesian jump from one model to another is what we have called creativity. Semiosis is then equivalent to creativity. The difference between Bayesian strategy and creative jump is the same as the difference between normal science and paradigm shift (Kuhn, 1962)

Let us discuss in detail the difference between a *type I cognitive task*, which implies changing hypothesis *h within a model*, that is, climbing a single mountain, and a type *II cognitive task*, which implies *changing model*, that is, jumping over to another mountain.

We formalize a model as a set of dynamical variables x_i $(i = 1, 2, \ldots, N)$, N being the number of degrees of freedom, with the equations of motion

$$\dot{x}_i = F_i(x_1, \ldots, x_N; \mu_1, \ldots, \mu_M) \tag{2}$$

Where F_i are the force laws and the M numbers μ_i represent the *control parameters*. The set $\{\boldsymbol{F}, \boldsymbol{x}, \boldsymbol{\mu}\}$ is the model.

Changing hypotheses within a model means varying the control parameters $\{\boldsymbol{\mu}\}$, as we do when exploring the transition from regular to chaotic motion within some model dynamics.

Instead, *changing code, or model*, means selecting different sets of degrees of freedom $\boldsymbol{y}$, control parameters ν and equations of motion $\boldsymbol{G}$ as follows:

$$\dot{y}_i = G_i(y_1, \ldots, y_R; \nu_1, \ldots, \nu_L) \tag{3}$$

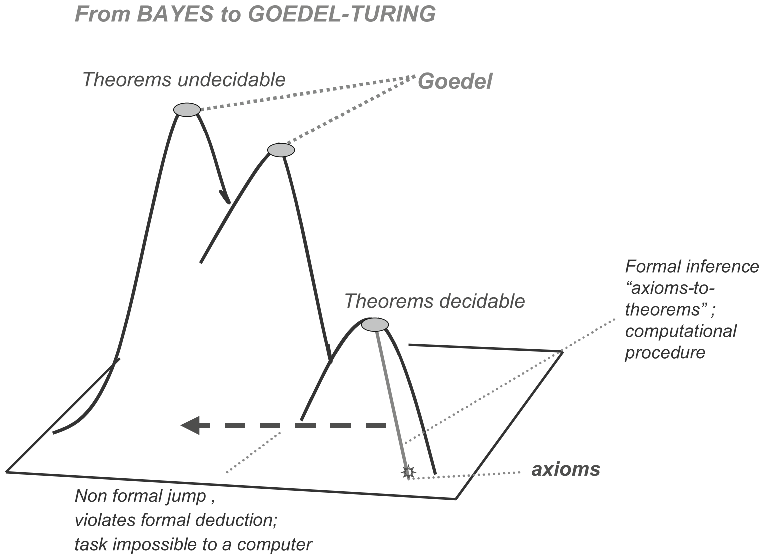


Fig. 9 Gödel's first incompleteness theorem (1931): For any consistent formal, computably enumerable theory that proves basic arithmetical truths, an arithmetical statement that is true, but not provable in the theory, can be constructed. "Provable in the theory" means "derivable from the axioms and primitive notions of the theory, using standard first-order logic". Computer equivalent (Turing1936 = Halting problem): a universal computer, for a generic input, cannot decide to stop

Where R and L are different respectively from N and M. The set $\{G, y, \nu\}$ is the new model.

While changing hypotheses within a model is an a-semiotic procedure that can be automatized in a computerized *expert system*, changing model implies catching the meaning of the observed world, and this requires what is called *embodied cognition* (Varela et al., 1991). Embodied cognition has been developed over thousands of generations of evolutionary adaptation, and we are unable so far to formalize it as an algorithm.

This no-go statement seems to be violated by a class of complex systems, which has been dealt with successfully by recursive algorithms. Let us consider a space lattice of spins, with couplings that can be either ferromagnetic or anti-ferromagnetic in a disordered, yet frozen way (spin glass at zero temperature, with quenched disorder). It will be impossible to find a unique ground state. For instance having three spins A, B, and C in a triangular lattice, if all of them have ferromagnetic interaction, then the ground state will consist of parallel spins, but if instead one (and only one) of the mutual coupling is anti-ferromagnetic, then there will be no satisfactory spin orientation compatible with the coupling (try with: A-up, B-up, C-up; it does not work; then try to reverse a single spin, it does not work either).

This model has a cognitive flavor, since a brain region can be modeled as a lattice of coupled neurons with coupling either excitatory or inhibitory, thus resembling a spin glass, (Hopfield, 1982; Amit et al., 1985; Toulouse et al., 1965). We have a large number of possible ground states, all including some frustration. Trying to classify all possible configurations is a task whose computational difficulty (either, program length or execution time) diverges exponentially with the size of the system. Sequentially related changes of code have been successfully introduced to arrive at finite-time solutions. (Mezard et al., 1987; Solomon, 1995).

Can we say that the mentioned solutions realize the reductionistic dream of finding a suitable computer program that not only climbs the single probability hill, but also is able to chose the best hill? If so, the optimization problem would correspond to understanding the *meaning* of the object under scrutiny.

We should realize however that spin glasses are frozen objects, given once for ever. A clever search of symmetries has produced a spin glass theory (Mezard et al., 1987) that, like the Renormalization Group (RG) for critical phenomena (Wil-

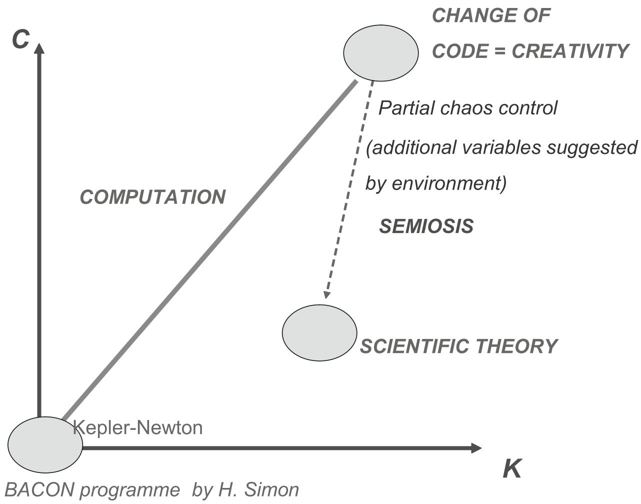


Fig. 10 C–K diagram (C = computational complexity; K = information loss rate in chaotic motion): Comparison between the procedure of a computer and a semiotic cognitive agent (say: a scientist). The computer operates within a single code and C increases linearly with K. A scientist explores how adding different degrees of freedom one can reduce the high K of the single-code description. This is equivalent to the control operation of Fig. 6; it corresponds to a new model with reduced C and K. The BACON program (Simon, 1980) could retrieve automatically Kepler's laws from astronomical data just because the solar system, approximated by Newton two-body interactions, is chaos-free

Table 1 Reduction of complexity by code change

1	Electricity; magnetism; optics	Maxwell equations
2	Mendeleev table	Quantum atom (Bohr, Pauli)
3	Zoo of 100 elementary particles	SU(3) – quarks (M Gell Mann)
4	Scaling laws in phase transitions	Renormalization group (K. Wilson)

son, 1983) discovers a recursive procedure for changing codes in an optimized way. Even though the problem has a large number of potential minima, and hence of probability peaks, a suitable insight in the topology of the abstract space embedding the dynamical system has led to an optimized trajectory across the peaks. In other words, the correlated clusters can be ordered in a hierarchical way and a formalism analogous to RG applied.

It must be stressed that this has been possible because the system under scrutiny has a structure assigned once for ever. In everyday tasks, we face a system embedded in an environment, which induces unpredictable changes in course of time. This rules out the nice symmetries of hierarchical approaches, and rather requires an adaptive approach. Furthermore, a real life context sensitive system has to be understood within a reasonably short time, in order to take vital decisions about it.

We find again a role of control of chaos in cognitive strategies, whenever we go beyond the limit of a Bayes strategy. We call *creativity* this optimal control of neuronal chaos.

Figure 10 sketches the reduction of complexity and chaos which results from a creative scientific step. Some examples of scientific creativity are listed in Table 1.

4 Conclusions

From what we have discussed, the search for meaning, i.e. a creative decision, is NON-Bayesian,

The world is complex, that is, not grasped by a unique model. How to choose among alternative models? There are two ways:

i) arbitrarily (kind of oracle), this amounts to ***relativism***;

or

ii) along some guidelines (fertility and latitude of the new explanation; useful outcomes); since any cognitive decision modifies (through our actions) the environment, we perform several trials with the aim of stabilizing the chaotic agent-environment dynamics. Thus, a ***semiotic approach*** to cognition discloses hard facts, it is a hint of an ***ontology***.

The same re-coding freedom inherent in the transient stabilization of chaos holds for ethical decisions. But behold, our freedom is CONDITIONAL on the environmental features in which we are embedded. Let me quote two philosophers:

– Aristotle: *thought and desire play a coordinate role* (Nicomachean Ethics),
– Vico: *poetic wisdom is the result of perception, memory and imagination* (The New Science).

Our freedom is by NO means ABSOLUTE as instead stated by

– Kant: *freedom = independency from cause-effect chains = not-caused cause* (Critique of Practical Reason).

The creative jump is not arbitrary, but guided by the situation in which we are embedded. We re-adjust the code until we find a satisfactory reading of the world around us.

Let us return to Fig. 9 but with a large number of peaks, to depict a complex situation (Fig. 11). The ascent to a single Bayes peak yields certainty, i.e. fidelity to a protocol, NOT truth but PAC (probably approximately correct) knowledge (Vidyasagar, 1987).

As we look for meaning, we jump non-algorithmically across several peaks, exploring a sizable region of a complex space; we speak of ***partial truth***, "*from a certain point of view*" (Agazzi, 1974).

This is the only truth accessible to a scientific program; it is a modern formulation of the old definition of truth as *adaequatio intellectus et rei* (Aquinas, 1992). ***Global truth*** is not accessible to us, as a complex universe is beyond our investigation power.

Would this mean a relativistic attitude? Not al all, since any starting point X has a pre-scientific motivation.

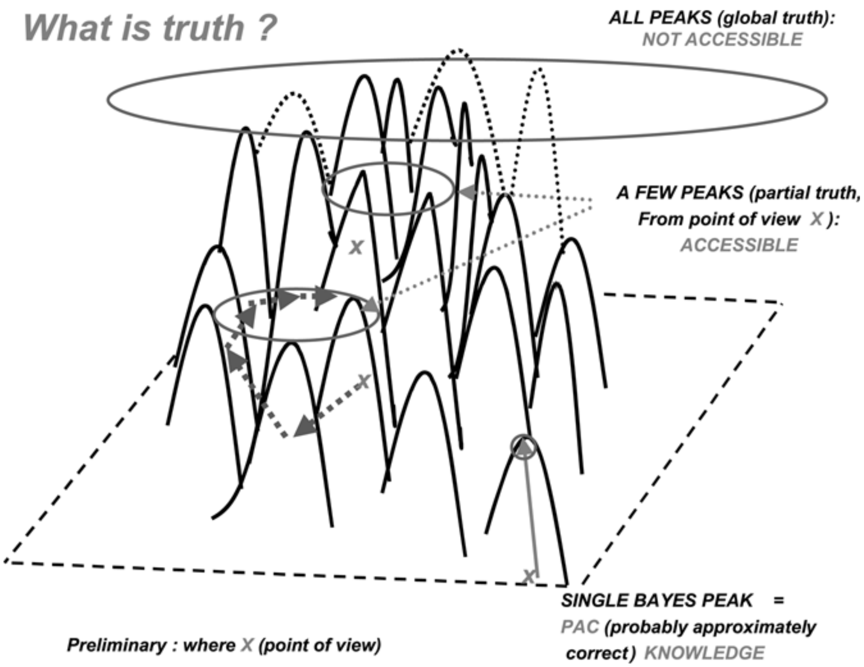


Fig. 11 Comparsison of three procedures: i) Climbing the single Bayes peak, leading to PAC Knowledge (what a computer does); ii) Different creative endeavors, starting from different X and exploring separate regions of a complex space = partial truth (what a creative mind does); iii) Exhausting all peaks of a complex problem, thus reaching a global truth (what nobody can do)

It corresponds to a different choice of an investigation area, thus it corresponds to an ethical, better to say, ethological motivation, depending on personal motivations. For instance, in selecting an area of physics, I have devoted my efforts to quantum optics and chaotic phenomena, staying away from elementary particles and cosmology. Once made clear my X, I can compare my results (my partial truth) with that of other colleagues.

As a final remark, we compare the *hermeneutic circle*, peculiar of cognitive agents (animals or robots) with a limited repertoire, with what I call the *hermeneutic spiral* (Fig. 12), which corresponds to the human language ability *to make an infinite use (in course of time) of finite resources*. The words' polysemy implies that we in general attribute several meanings or connotations to the same word. If these connotations are frozen once for ever, as in a historical dictionary, then we are in the upper case.

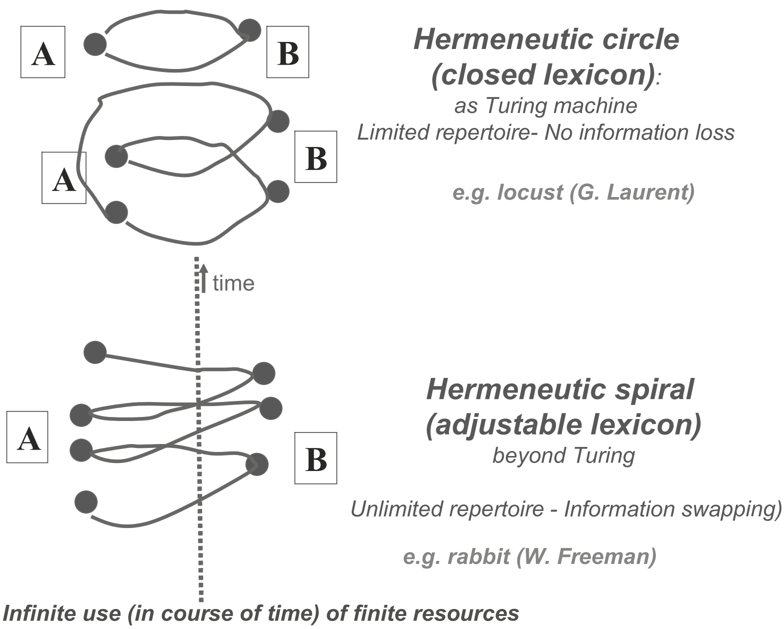


Fig. 12 Difference between hermeneutic circle and hermeneutic spiral. Let us consider an algorithm which takes from an item A to an item B. It the items are words of an ordinary language, they are polysemic, that is both A and B do not have a unique connotation. Selecting a specific connotation of A the algorithm arrives at a specific connotation of B. By reversing the algorithmic procedure, one returns at the original connotation of A either in one loop (uppermost figure) or in a finite number of loops (central figure).These cases correspond to a cognitive agent with a limited repertoire and no information loss. If, as decribed in non-Bayesian cases, there information swapping, i.e. re-coding, the number of connotations of the words A and B is for all purposes unlimited. As time flows, there is a semiotic re-adjustment between cognitive agent and environment which yields new connotations for the same lexical terms, as it occurs in poetry and music

Suppose a grammatical operator takes from a sub-meaning of the word A to a sub-meaning of B; applying the inverse operator we recover the initial connotation of A, either in one or several steps. A neurobiological example is offered by the olfactory system of the locust (Laurent et al., 1996). Exposed to a "cherry " odour, it responds with a specific sequence of neuronal spikes ; re-exposing the locust to the same odour at a later time, the odour is encoded by the same sequence.

If instead we do the experiment with a more sophisticated animal as a rabbit (Freeman, 1991), any successive exposure to the same odour is encoded in different spike patterns. The rabbit "feels" that some time has passed by and that it is experiencing the same perception but not for the first time. This spiral along time, whereby we never repeat exactly the same connotation, is peculiar of human creativity, in language (we make new poetry with the same lexicon) or music (we make new pieces with the same notes). Indeed, it is our common experience that revisiting after a while a piece dear to us, e.g. a Bach Cantata, we discover new meanings, since our inner universe has grown in the meantime.

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* Since the topics of this review track my research line along the years, quite a few considerations can be found scattered in my research papers. My publications can be downloaded from my homepage: www.inoa.it/home/arecchi, at: List of publications – Research papers in Physics.

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Complexity Theoretic Bounded Rationality and Satisficing*

Kumaraswamy Vela Velupillai

Dedicated to the Memory of Massimo Salzano
Scholar and Friend†

Abstract. Formally, the orthodox rational agent's 'Olympian' choices ([9], p. 19) appear to be made in a static framework. However, a formalization of consistent choice, underpinned by computability, suggests *satisficing* in a *boundedly* rational framework is not only more general than the model of 'Olympian' rationality; it is also consistently dynamic. This kind of naturally process-oriented approach to the formalization of consistent choice can be interpreted and encapsulated by varieties of frameworks of *theories of complexity*.

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* A title that might be more appropriate would have been: *Boundedly Rational Choice and Satisficing Decisions – Computable Foundations for a Complexity Theoretic Approach.* This, while reflecting the main thrust of the contents of the paper, and its attempt at aiming to meet the stated scope of the event, is obviously too long. Hence the truncated, if also slightly less informative, title. Most importantly, however, the longer title would have signalled, explicitly, the way I am trying to meet Massimo Salzano's abiding interests in the triptych of *Bounded Rationality*, *Satisficing* and *Complexity* – the latter, in particular, in all its many variations and formal splendour. I think it is my own long-standing interest and work in *computable economics* that first brought them to Massimo's attention. Thus, the foundations in the title refers to those provided by *computable economics*. But I want to emphasize that *computable* here refers to both *recursion theoretic* and *constructive* aspects of that concept. This shift in trying to be more inclusive came about by my increasing discomfort with the strictures of the *Church-Turing Thesis* – a stricture that is nonbinding on the *Brouwer-Bishop variant of constructive mathematics*. My own adherence is to this variant of constructive mathematics, with a distinct slant towards the former, for its philosophical basis in *intuitionism*. I am confident Massimo Salzano would have approved my stance.

† I am deeply indebted and immensely grateful to *Marisa Faggini* for the kind invitation and generous support - both intellectual and material. The meticulous logistics, sensitive precision and unmatched warmth with which the event was organised was most gratifying to experience, enjoy – and remember.

M. Faggini, C.P. Vinci (Eds.), *Decision Theory and Choices: a Complexity Approach*

Massimo Salzano – Friend and Scholar:

We are all tenants of time –
its arrow mercilessly cruel.
Your tenancy was needlessly cut short –
not your generous legacy.
We try to ride that cruel arrow –
a part of the way,
softening it with your rich intellectual legacy.
It cushions the pointed arrow –
and blunts its pointless cruelty.
I hope our softened tenancy
lights an intellectual candle – or even two,
to celebrate your rich heritage.

Vela Velupillai, 4 January, 2009

1 *Homage* to Salzano – A Preamble

> "It is ... well-known that: (a) economic systems are characterised by *complex dynamics* and ... (b) when the system is complex it is *not always* describable *in an analytical form*, or, at least, such a description proves of little use."
>
> Salzano (cf., [4], p. 45; italics added).

In this concise, pungent, assertion, Massimo Salzano, with admirable directness got to the point of many of the conundrums posed by complex systems, however characterized. The care with which Massimo Salzano qualified the loose description of a complex system – '*not always* describable in an *analytical form*' (italics added) – shows that he had a fine, honed, intuition for what might be required to make the characterization rigorous. The immediate question one might pose is whether it is possible to specify, in an *effective* way, those complex dynamic systems that are characterizable in an analytical form. Equally importantly, Massimo Salzano conflates, correctly in my view, 'structure' and 'dynamics' when he refers to 'economic systems' being 'characterized by 'complex dynamics'. This is a vision I first learned to appreciate as a student of Richard Goodwin's lifelong attempts to analyze structure, *linearly* formulated, with its implied dynamics, *nonlinearly* formalized.

The theme of this conference – in honour of Massimo Salzano – is, however, '**Decision Theory and Choice: a Complexity Approach**'. No one person combined and encapsulated better, in an intrinsically *complex dynamic* framework, a *computationally founded*[1] theoretical system of *choice* and *decision*, both entirely rational in a broad sense, than Herbert Simon. In this contribution I try, therefore, to pay *homage* to Massimo Salzano, by standing on the shoulders of Herbert Simon, in fairly precise and formal ways. In a nutshell, the aim is to reformulate, with textual support from Herbert Simon's characterizations and suggestions, bounded rationality and satisficing in a computable framework so that their intrinsic complex dynamics is made explicit in as a straightforward way as possible. To achieve this aim, in the tradition of Simon, I start from orthodox underpinnings of rational choice theory and extract its inherent procedural content, which is usually submerged in the inappropriate mathematics of standard real analysis.

In the next section, some substantiation for 'standing on Simon's shoulders' will be outlined. On the basis of Simon suggestions given in §2, I go on, in §3, to outline the kind of formalism that provides computable foundations for a complexity approach to decision theory and choice, both rationally conceived. In §4, suggestions on the formal machinery that can be built, to make explicit the kind of dynamic and computational complexities intrinsic to the computable foundations of decision and choice, are given. A brief concluding section 5, summarizes the results and ends with brief signposts towards the care that must be taken in assertions about bounded rationality and satisficing as special cases of, or constrained versions of, the orthodox formalisms.

Several important background caveats on the mathematical underpinnings of the computable methodology with which I approach the issues tackled in this paper must be pointed out, at the very outset – lest the unwary or un-honed (in algorithmic mathematics) reader concentrates on inessentials. The main complexity concept I shall ultimately be interested in, for rationally conceived decisions and choices, is *computational complexity* (although the kind of *dynamic complexity*, associated with formal dynamical systems, that also will be discussed, can be '*reduced*' to formal computational complexity).

Computational complexity theory is doubly related to mathematical economics and economic theory: first, as a theory of the *efficiency of computations* it is best viewed as the *economic theory of computations*; secondly, in having at its central core the paradigmatic combinatorial, intractable, *NP-Complete*, *Travelling Salesperson's Problem* (TSP). In the former case, it must first be remembered that the pure theory of computations abstracts away from all kinds of *resource constraints*. Computational complexity theory, the 'applied' theory of computation, is its finessing, taking explicit account of resource constraints, typically time and space con-

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[1] 'Computational' has always meant 'computable' in the Turing sense, at least in my reading of Simon's magisterial writings. In particular, in the context of bounded rationality, satisficing and their underpinnings in the architecture of human thinking, it was the path broached by Turing that guided Simon's pathbreaking contributions. In a volume celebrating '*The Legacy of Turing*' ([11], p. 81 & p. 101), Simon's essay, *Machine as Mind*, began and ended as follows:

> "The title of my talk is broad enough to cover nearly anything that might be relevant to a collection memorializing A.M. Turing. . . . If we hurry, we can catch up to Turing on the path he pointed out to us so many years ago."

straints. One of the modern pioneers of computational complexity theory, Richard Karp, perceptively noted, [1], p. 464, italics added:

> "[I] do think there are some very worthwhile and interesting analogies between complexity issues in computer science and in economics. For example, economics traditionally assumes that the agents within an economy have universal computing power and instantaneous knowledge of what's going on throughout the rest of the economy. Computer scientists deny that an algorithm can have infinite computing power. *They're in fact studying the limitations that have arisen because of computational complexity. So, there's a clear link with economics*."

Unfortunately, where even this generous analogy is misleading is in assuming that 'economics traditionally assumes that the agents within an economy have universal computing power.' In fact, not even this fantastic assumption is explicitly made 'in economics' (unless it is of the Simonian variety of behavioural economics). This is why it is important to be aware that in computational complexity theory, the characterizing framework is one of *problem solving*, with *a model of computation* explicitly underpinning it, as *decision problems*.

Now, a *decision problem* asks whether there exists an *algorithm* to *decide* whether a mathematical assertion does or does not have a proof; or a formal problem does or does not have a solution. Thus the characterization makes clear the crucial role of an underpinning model of computation; secondly, the answer is in the form of a *yes/no* response. Of course, there is the third alternative of '*undecidable*', too, but that is a vast issue outside the scope of this paper. It is in this sense of *decision problems* that I shall interpret the word 'decisions' in the conference title.

As for 'problem solving', I shall assume that this is to be interpreted in the sense in which it is defined and used in the monumental classic by Newell and Simon ([2]).

Decisions, in the computational and *problem solving* tradition of Herbert Simon, have a more general and fundamentally different characterization in computable economics.

Finally, the *model of computation*, in the above senses and contexts, is the *Turing model*, subject to the *Church-Turing Thesis*. I shall adhere to this tradition, but – at least for my results and propositions – this is only for convenience; I believe all my formal results can also be derived without assuming the ChurchTuring Thesis, hence within the formalism of constructive mathematics.

2 Standing on Simon's Shoulders[2]

> "The standard theoretical approach used in economics is one loosely based on a vision of rational agents optimizing, and is a consideration of how a system composed of such optimizing agents would operate. . . .
>
> . . .

[2] My first attempts at trying to make the case for boundedly rational, adaptive behaviour and satisficing, in solving decision problems in a computable framework, were made in Chapter 4 of *Computable Economics* ([12]). To the best of my knowledge, no other work makes this point – whether in a computable framework, or not.

> The complexity approach does not accept that, and proposes a different vision, which is a *generalization of the traditional analysis*."
>
> Salzano and Colander (cf., [5], p. IX; italics added)

In this section I shall try to provide a 'Simonian context' for the way I aim to tackle the problem of a 'complexity approach' to 'decisions and choice'. This is provided by means of two extensive 'quotations' – one, from a long letter Herbert Simon wrote me, in May, 2000; and the other, from one of his classic pieces. They make explicit his visions of *complexity*, based on the *Turing model of computation* and the nature of the way internal and external constraints determine *satisficing* in a *boundedly rational* context. I proceed in this unconventional way simply to make it clear, from the outset, that my own vision is that a boundedly rational agent satisficing by implementing (rational) decisions is the general case; the Olympian model of rational choice is the special case.

On May 25th, 2000, Herbert Simon wrote me as follows (referring to having read my book on *Computable Economics*, [12]; emphases added):

> I want to share some first impressions on my reading of "Computable Economics." ... I was delighted and impressed by the mileage you could make with Turing Computability in showing how nonsensical the Arrow/Debreu formulation, and others like it, are as bases for notions of human rationality. Perhaps this will persuade some of the formalists, where empirical evidence has not persuaded them, of what kinds of thinking humans can and can't do – especially when dealing with the normative aspects of rationality.
>
> ...
>
> As the book makes clear, my own journey through bounded rationality has taken a somewhat different path. Let me put it this way. There are many levels of complexity in problems, and corresponding boundaries between them. Turing computability is an outer boundary, and as you show, any theory that requires more power than that surely is irrelevant to any useful definition of human rationality. A slightly *stricter boundary is posed by computational complexity*, especially in its common "worst case" form. We *cannot expect people (and/or computers) to find exact solutions for large problems in computationally complex domains*. This still leaves us far beyond what people and computers actually can do. The next boundary, but one for which we have few results except some of Rabin's work, is *computational complexity for the "average case"*, sometimes with an "almost everywhere" loophole. That begins to bring us closer to the realities of real-world and real-time computation. Finally, we get to the empirical boundary, measured by laboratory experiments on humans and by observation, of the level of complexity that humans actually can handle, with and without their computers, and – perhaps more important – what they actually do *to solve problems that lie beyond this strict boundary* even though they are within some of the broader limits.
>
> ...
>
> The latter is an important point for economics, because *we humans* spend most of our lives *making decisions that are far beyond any of the levels of complexity we can handle exactly*; and *this is where satisficing*, floating aspiration levels, recognition and heuristic search, and similar *devices for arriving at good-enough decisions*[3] *take over*. A parsimonious economic theory, and an empirically verifiable one, shows how *human beings, using very simple procedures, reach decisions that lie far beyond their capacity for finding exact solutions by the usual maximizing criteria*

[3] The famous Voltaire precept comes to mind: '*The perfect is the enemy of the good*'!

> ...
>
> So I think we will continue to proceed on parallel, but somewhat distinct, paths for examining *the implications of computational limits for rationality* – you the path of mathematical theories of computation, I the path of *learning how people* in fact *cope with their computational limits*. I will not be disappointed however if, in the part of your lives that you devote to experimental economics, you observe phenomena that seduce you into incorporating in your theories some of these less general but very real departures from the rationality of computational theory. This seems to me especially important if we are to deal with the mutual outguessing phenomena (shall we call them the Cournot effects?) that are the core of game theory.
>
> I am sure that you will be able to interpret these very sketchy remarks, and I hope you will find reflected in them my pleasure in your book. While I am fighting on a somewhat different front, I find it greatly comforting that these outer ramparts of Turing computability are strongly manned, greatly cushioning the assault on the inner lines of empirical computability.

Several important issues are clarified by Simon in these elegant observations. First of all, the defining – and decisive – role played by the *Turing model of computation* as the benchmark for his own fundamental work on *computationally underpinned work on rationality* – i.e., bounded rationality – and satisficing decisions. Secondly, it is also unambiguously clear that the various boundaries delineated and defined by *computational complexity theory* – based, of course, on the Turing model of computation – are with reference to *the problems* that boundedly rational agents try to solve – i.e., the level of complexity is that which is defined by the nature of the problem to be solved, not determined *solely* by the complexity of the computational architecture of the boundedly rational agent. Thirdly, boundedly rational agents actually do solve 'problems that lie beyond the strict boundary' of formally feasible, computationally solvable, problems. The hint may well be that boundedly rational agents do discover, by heuristic means, methods to satisfactorily solve problems that computational complexity theory places beyond the empirically feasible range. To the extent that computational complexity theory is underpinned by a model of computation, formal complexity boundaries are defined for the degrees of solvability of computable problems; uncomputable problems are beyond the 'outer boundary'. Fourthly, and perhaps most importantly, boundedly rational agents actually solve decision problems, in a satisficing framework, that lie beyond the orthodox domains of solvability – perhaps the best way to state this is that *Olympian means and aims* are not capable of solving the problems framed within the *Olympian model of rational choice*. The key to interpret this important observation by Simon is to note that the traditional, *half-naked*, framework of 'optimization' is replaced by the fully-clothed one of *decision problems*. The *half-naked* nature of the Olympian framework is due to the absence of a 'model of computation' to underpin its formalization – and that, in turn, is almost entirely due to the unfortunate reliance of the mathematics of real analysis of a very truncated sort. This is the sort that is founded on set theory, with its uncomputable and non-constructive handmaiden, the axiom of choice.

The above characterisations and comments are further strengthened by the following, even more explicit, commentaries by Simon, on the distinction between the internal and external constraints going into the definition of a boundedly rational agent's confrontation with a decision problem in a satisficing framework:

> "Now if an organisms is confronted with the problem of behaving approximately rationally, or adaptively, in a particular environment, the kinds of simplifications that are suitable may depend not only on the characteristics – sensory, neural, and other – of the organism, but equally upon the structure of the environment. Hence, we might hope to discover, by a careful examination of some of the fundamental structural characteristics of the environment, some further clues as to the nature of the approximating mechanisms used in decision making.
>
> ...
>
> [T]he term environment is ambiguous. I am not interested in describing some physically objective world in its totality, but only those aspects of the totality that have relevance as the 'life space' of the organism considered. Hence, what I call the 'environment' will depend upon the 'needs,' 'drives,' or 'goals' of the organism and upon its perceptual apparatus."
>
> [8],p. 21

The point, again, is *not* that the theoretical analyst is concerned with 'absolute' constraints – either of the internal structure of the decision making entity, or of the external environment of which a problem is a part – and in which it is embedded. The relevant architecture of the decision making entity, in this case that of a *computationally conceived rational economic agent*, solves a decision problem embedded, and emerging from, an environment, also computationally underpinned. The approximations are two-pronged: one, on the architecture of the computationally conceived rational agent – i.e., the boundedly rational agent; the other, on the computationally underpinned environment, now conceived within the satisficing framework of a decision problem. This does not entail, in any way at all, that the approximations of a computationally conceived agent is a special case of the orthodox rational agent in the Olympian mode of choice. Nor does it imply at all that the approximation of the decision problem in the satisficing framework is a special case of the Olympian model of indiscriminate optimization. The numerous attempts, claiming to be within a behavioural economics setting, because, for example, the agents are supposed to be boundedly rational *fail in the former sense*; i.e., assuming that the agent in such allegedly behavioural settings are boudedly rational because they are assumed to be constrained – for example by having only 'limited' memory, modelled as finite automata, rather than as Turing machines – versions of the Olympian agent. As for an example of the failure from the point of view of the second 'vision' – regarding the approximations on, and of, the environment, the canonical example is, of course *the folly of considering an integer linear programming problem as a special case of the standard linear programming problem.*

In fact, this will be the illustrative example I shall choose for my formal description and discussion of these distinctions, so as to find a way to state and define the case for the vision that places the boundedly rational agent in a satisficing setting to solve a decision problem as the general one – and the Olympian model as *a special, and uninteresting, case*.

3 Bounded Rationality as a Superset of Olympian Rationality

> [Linear Programming problems are] solvable in polynomial time. In 'Integer Linear Programming' we come to a field where the problems in general are less tractable, and are *NP-Complete*. It is a general belief that these problems are not solvable in polynomial time. The problems in question are:
>
> - solving systems of linear diophantine inequalities, i.e. solving linear inequalities in integers;
> - solving systems of linear equations in nonnegative integer variables;
> - solving *integer linear programming* problems.
>
> [T]hese three problems are equivalent in the sense that any method for one of them yields also methods for the other two. Geometrically, the problems correspond to the intersection of a lattice and a polyhedron."
>
> Schrijver ([7], pp. 2–3; italics in the original)

The simple analogy I wish to appeal to, for substantiating the case that the Boundedly Rational Agent is the general case and the Olympian Agent is the special case, is in terms of the classic difference between Integer Linear Programming and Linear Programming. *From the point of view of problem solving, underpinned by a model of computation*, the former is unambiguously the more general and the more *complex* case; the latter is the less general, *simpl*e case. It must also be emphasized that 'more complex' refers to the precise sense of computational complexity – as made clear by reference to *NP-Complete* in the above quote.

Consider the following abstract version of a formalization of what may be called the standard economic optimization problem (*SEP*):

$$\begin{array}{lll} \text{Minimize} f(x) & & \\ \text{subject to:} & g_i(x) \geq 0, & i = 1, 2, \ldots, m \\ \text{and:} & h_j(x) = 0, & h_j = 1, 2, \ldots, p \end{array}$$

[Naturally, with standard – i.e., 'convenient but irrelevant' – assumptions on f, g and h].

Now, consider the following variant of *SEP*:

Definition 1 *SEP*:*

An optimization problem is a pair $\{F, c\}$, where:

F: the set – the domain – of possible alternatives;

$c : F \to \Re$ (e.g., the criterion function);

Then the *problem to solve*, associated with *SEP* is*: Find $f \in F$ such that $c(f) \leq c(g), \forall g \in F$.

Now, make explicit the computational content of an *SEP*∗ as:

Definition 2 *SEP*^{TM}

- Given a *combinatorial object* (i.e., a number-theoretically specified object) f and a set of parameters, S, *decide* whether $f \in F$ (where F is characterized by S).

- Assume that this *decision procedure* is executed by algorithm T_f (standing for the *Turing Machine* indexed by f, which has been *effectively* encoded, number-theoretically).
- After the decision implemented by T_f use another (*algorithmic*) decision procedure to *compute* the value $c(f)$, where c is characterised by the set of parameters Q. Call this latter decision procedure T_c.
- Note that S and Q are to be represented number-theoretically – for example, *Gödel-numbered.*

Remark 3 *Firstly, to start with a 'given combinatorial object' ab initio is part of the claim to generality of the decision problem approach to problem solving in the satisficing, boundedly rational, vision. Secondly, the combinatorial object is encoded number theoretically to be processed by a model of computation. Simon does not always assume that the human problem solver is endowed with the full facilities of the most powerful model of computation (subject to the Church-Turing Thesis), but limited by various psychological and neurologically determined and informed factors. It is in this step that the qualification* ***limited*** *or* ***bounded*** *gets its full significance in a problem solving context. Satisficing, however, comes together with the* ***decision problem*** *approach to problem solving, i.e., in the third of the above four step scheme. Finally, approximating the combinatorial object suitably, by the agent or the problem solver, is the step where the structure of the environment [8] comes into play.*

Now, consider the standard integer linear programming problem $(SLIP)$ as an example of *SEP*^{TM} :

Minimize $c'x$ *such that* $Ax = b$ and $x \geq 0$ & (possibly also c, b and A) $\in \mathbb{N}$ (the variables are, naturally, vectorial of suitable dimensions).

According to the *SEP*^{TM} interpretation this means:

- The parameters S, for the decision procedure T_f are given by A, b.
- Given any integer (vector) x, T_f *decides* whether $Ax = b$ and $x \geq 0$ are simultaneously satisfied.
- 'Then', T_c is implemented, which has c for Q to evaluate $c'x$ for each x decided by T_f.

Remark 4 *'Then', in the third step above, does not necessarily imply sequential actions by TMs. More complex decision tasks, employing richer varieties of SEP*^{TM} *could imply a set of TMs operating on a parallel architecture and executing decisions both synchronously and asynchronously. However, Simon almost invariably worked within a sequential, synchronous, framework – although he was, of course, quite familiar with the richer relative possibilities of parallel architectures.*

The two main conclusions of this section are the following. Firstly, given the computational underpinning of a problem solving approach to rational decision making and, therefore, the necessity of a model of computation to implement a decision problem, every such process has an intrinsic complexity measure in terms of computational complexity theory – in general in the form of *NP-Completeness*.

Secondly, the whole set up is naturally more general than the setting in which the Olympian Model is framed and formalized.

4 Computable Rational Agents and Satisficing

> "The theory proclaims *man to be an information processing system*, at least *when he is solving problems*. ...
>
> ...
>
> An information processing theory is dynamic, ..., in the sense of describing the change in a system through time. Such a theory describes the time course of behavior, characterizing each new act as a function of the immediately preceding state of the organism and of its environment.
>
> The natural formalism of the theory is the program, which plays a role directly analogous to systems of differential equations in theories with continuous state spaces ...
>
> All dynamic theories pose problems of similar sorts for the theorist. Fundamentally, he wants to infer the behavior of the system over long periods of time, given only the differential laws of motion. Several strategies of analysis are used, in the scientific work on dynamic theory. The most basic is taking a completely specific initial state and tracing out the time course of the system by applying iteratively the given laws that say what happens in the next instant of time. *This is often*, but not always, *called simulation*, and is one of the chief uses of computers throughout engineering and science. It is also the mainstay of the present work."
>
> Newell & Simon ([2], pp. 9–12; italics added)

The point here is that a (rational) problem solving entity is considered to be an information processing system, which is intrinsically dynamic, encapsulated in the 'program' and, hence, naturally analogous to the role played by, say, 'differential equations', in classical dynamics[4]. With this in mind, and against the backdrop provided by the discussion in the previous section, the strategy for my formalization exercise can be summarized in the following sequence of steps:

- Extract the procedural content of orthodox rational choices (in theory).
- Formalize such a procedural content as a process of computation.
- Given the formalized procedural content as a process of computation, to be able to discuss its computational complexity.
- Show the equivalence between a process of computation and a suitable dynamical system.
- To, then, show the possibility of non-maximum rational choice.
- Then, to show that such behaviour is that which is manifested by a boundedly rational, satisficing, agent.

4.1 Rational Choice as a Computation by a Universal Turing Machine

> "In situations that are complex and in which information is very incomplete (i.e, virtually all real world situations), the behavioral theories deny that there is any magic for producing

[4] Indeed, even more so in modern dynamical systems theory, particularly in its post-Smale varieties.

> behavior even approximating an objective maximizing of profits and utilities. They therefore seek to determine what the actual frame of the decision is, *how that frame arises from the decision situation*, and *how*, *within that frame*, *reason operates*.
>
> In this kind of complexity, there is no single sovereign principle for deductive prediction. The *emerging laws of procedural rationality* have much more the complexity of molecular biology that the simplicity of classical mechanics."
>
> Simon ([10], p. S223, italics added)

The following result encapsulates, formally, the content of the first three steps of the above six-step scheme:

Theorem 5 *The process of rational choice by an economic agent is formally equivalent to the computing activity of a suitably programmed (Universal) Turing machine.*

Proof. By construction. See §3.2, pp. 29–36, *Computable Economics* [12] ■

Remark 6 *The important caveat is 'process' of rational choice, which Simon – more than anyone else – tirelessly emphasized by characterizing the difference between 'procedural' and 'substantive' rationality; the latter being the defining basis for Olympian rationality, the former that of the computationally underpinned problem solver facing decision problems. Any decision – rational or not – has a time dimension and, hence, a content in terms of some process. In the Olympian model the 'process' aspect is submerged and dominated by the static optimization operator, By transforming the agent into a problem solver, constrained by computational formalisms to determine a decision problem, Simon was able to extract the procedural content in any rational choice. The above result is a summary of such an approach.*

Definition 7 *Computation Universality of a Dynamical System*

A dynamical system – discrete or continuous – is said to be capable of computation universality if, using its initial conditions, it can be programmed to simulate the activities of any arbitrary Turing Machine, in particular, the activities of a Universal Turing Machine.

Lemma 8 *Dynamical Systems capable of Computation Universality can be constructed from Turing Machines*

Proof. See [12] and [15] ■

Theorem 9 *Only dynamical systems capable of computation universality are consistent with rationality in the sense that economists use that term in the Olympian Model.*

Proof. See pp. 49–50, [12]. ■

Remark 10 *This result, and its proof, depend on the first theorem in this sub-section and, therefore, its background basis, as explained in the Remark following it, given above. In this way, following Simon's vision as outlined in the opening quote of* *this section, the definition of rationality is divorced from optimization and coupled to the decision problems of an information processing problem solver, emphasizing the procedural acts of choice.*

Theorem 11 *Non-Maximum Rational Choice*

No trajectory of a dynamical system capable of universal computation can, in any 'useful sense' (see Samuelson's Nobel Prize lecture, [6]), be related to optimization in the Olympian model of rationality.
Proof. See [13]

Theorem 12 *Boundedly rational choice by an information processing agent within the framework of a decision problem is capable of computation universaltiy.* ■

Proof. An immediate consequence of the definitions and theorems of this subsection. ■

Remark 13 *From this result, in particular, it is clear that the Boundedly Rational Agent, satisficing in the context of a decision problem, encapsualtes the only notion of rationality that can 'in any useful sense' be defined procedurally.*

The above definitions, theorems and lemma give formal content to the six-point formalization strategy outlined at the beginning of this section.

5 Concluding Notes

> "In your opening chapter, you are very generous in crediting me with a major role in the attention of the economics profession to the need to introduce limits on human knowledge and computational ability into their models of rationality. ... But you seem to think that little has happened beyond the issuance of a manifesto, in the best tradition of a Mexican revolution."
>
> Herbert Simon to Ariel Rubinstein [[3], p. 189].

To give a complexity theoretic basis for bounded rationality and satisficing it is necessary to underpin them in a dynamic model of choice in a computable framework. This I believe has been achieved in this paper, in a setting that is entirely faithful to Herbert Simon's precepts and lifelong decision-theoretic research program. A by-product of the results in this paper is the exploitation of the duality between dynamical systems and computability. With this duality it was possible to show in what sense bounded rationality is the more general case, in the case of an information processing problem solver, set in the context of a decision problem, and the Olympian model is the special case.

A rational choice framework that is entirely underpinned by computability and dynamical systems theory is naturally amenable to complexity theoretic analysis – both in terms of standard computational complexity theories and the more vague dynamic complexity theories. In other, companion writings (see [14] & [15]), I have developed these two themes in much greater detail and, due to the natural limitations of space in a publication of this kind, I shall have to refer the interested reader to them for further developments and the many felicitous connections.

Most importantly, I hope the message in this paper disabuses unscholarly assertions about bounded rational behaviour being a case of approximations to, of constrained versions of, the Olympian Model and satisficing, concomitantly, a case of sub-optimal decision process. These kinds of unscholarly assertions permeate every strand of experimental economics and game theory. For example, in the case of games played by automata, bounded rationality is modelled in terms of finite automata, ostensibly to take into account 'limited' memory as one case of constrained Olympian rationality. Nothing in the Olympian model has anything to do with any kind of model of computation. How, then, can a special case of that become a model for computation by a finite anutomaton? Aimilar series of misguided examples can be cited from standard experimental economics – i.e., non-Game Theoretic experimental economics, not to mention orthodox choice theory.

Massimo Salzano's intuitive perception of the importance of complexity theoretic underpinnings for choice theory had dynamic orgins. In this essay paying, homage to an original and generous scholar, who shared my fidelity to Simon's research program on decision theoretic modelling or problem solvers, I have tried to expand that dynamic vision with its dual: the computable one. Together – the dynamic and the computable – combine to produce a ferociously complex framework, when implemented purely theoretically. But, mercifully, serious empirical investigations – of the kind Simon routinely practised all his life – entails judicious simplifications, as indicated in that letter from Simon to me, from which I quoted at the beginning of this essay.

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Optimisation and “Thoughtful Conjecturing” as Principles of Analytical Guidance in Social Decision Making*

Sergio Bruno

Abstract. The XXth can be considered the Century of the culture of optimisation, that has been outstandingly improved and applied. There is a difference, however, between applying it to complicate technical problems, such as the war logistics, or to complex social choices, where the identification of measurable objectives and their relative weights cannot be objective and stable through time. Ex ante evaluation of actions cannot be considered as a task having an objective or scientific nature, since a part of it depends on forecasting future events, the effectiveness of which depends only on elements of systemic inertia. Despite its non-scientific basis, the use of evaluation criteria may improve, in a probabilistic sense, the quality of decisions, since, as a procedure, it contributes to confer order to the imagination of the decision makers, to reflect in qualitative terms about possible futures, to communicate, to reach consensus. Its wide use tends to produce routines, that on the one side contribute to stabilise the environment and to make it forecastable but, on the other side, tend to hinder innovative projects. Still it remains basically an art, while often it disguises itself as being objective. The associated risks are analysed in detail.

1 Introduction

Today any important organisational decision has, in principle, evaluation and optimisation as its pre-requisites. The XXth could be labelled as the “Century of Optimisation”. Starting from prior sparse seeds, the “culture of optimisation” had become pervasive within the end of the Century. Terms such as “efficiency” and “effectiveness” sound good almost everywhere in the world, enjoy a high degree of social

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acceptance, independently of specific domains of application. Techniques of optimisation are applied universally, from war and peace logistics to engineering projects, from telecommunication to space activities, from environmental evaluations to social and economic choices.

It makes a substantial difference, however, whether optimisation involves actions affecting human beings and their interrelations, or instead problems in which simply material events, effects and interactions prevail. In the latter sphere complication prevails on complexity, laboratory experiments and tests can be made in most circumstances, knowledge about involved phenomena is more robust, it is easier to single out the objectives that are pursued and obtain agreement about them and their relative importance (the determination of what is usually identified as the "objective(s) function", OF). In the former one, instead, human aspirations and actions and their changing relationships are involved, and are involved not only in the act of a single choice, but also in the preparation and in the implementation of sequences of choices; the roles of the subjects involved in the choices, the quality and quantity of their knowledge are highly differentiated; those who make the choice are often different from those who are affected by it; the identification of the goals is highly problematical and the goals are not necessarily stable through time; experiments can be seldom performed; ideologies and interests play an important role, often spilling over theoretical models.

These important differences should have suggested very prudent attitudes in the attempts to extrapolate the rationale and the methods of optimisation to the sphere of social and economic problems, while not necessarily rejecting them, at least as a purely logical frame of reference. Quite on the contrary, by adopting a sort of "imperialistic" attitude, scientific communities have pushed for the generalisation of the optimisation techniques (and their "collaterals", such as the evaluation techniques) to most problems in the economic and social domain. At the same time these techniques have become widely accepted in our societies. The implied puzzle is thus twofold: is this diffusion acceptable and, if or when it is not, why is it accepted?

The community of economists has important responsibilities in this diffusion, not only because it has been the first community to aim at constructing for itself a scientific statute comparable to that of physicists, extending the logics of mechanical "laws" to the treatment of human affairs, but because of the very same "imperialism" embodied in the definition of economic "science". According to Robbins' definition, indeed, all the manifestations of what I called "culture of optimisation" are economic applications. If in fact economics is regarded as the scientific discipline which studies the *relationship* between goals and scarce means, independently of the nature of the ends and the means, any type of constrained maximisation belongs to it. Independently of the acceptance of this hyper-comprehensive definition and despite the important differences[1] between the different approaches to the problems of decision concerning social sub-sets, the "economical" halo is likely to have

[1] No doubts that the marginalist approach, since its first founding steps, relied on the tools of constrained maximisation. The economists, however, were interested in the *interactions* among rational individual optimisers and on their resulting equilibriums. Because of this, economists did not pay attention on how individuals determine their objectives, which they are and how are they or-

contributed to the way our modern societies perceive the messages stemming from the optimisation culture. They are perceived, broadly speaking, as being "scientific", at the worst as being "technical", in any case as being "rational"[2], "neutral", "objective".

Such perceptions, widely shared in both, the scientific communities and among actual choice-makers, are in my view wrong; they are wrong for many reasons but mainly because they underestimate the role of time in the decision-making processes. The "decision" is an act in an instant of time which *cuts off* (this is the etymology the word) at the same time a "process in time" (a long lasting phase during which alternatives and their likely outcomes are conceptually structured, in other words the choice problem is gradually defined and framed) and the neglected alternatives. The outcomes follow the decision in historical time and depend on the relationships between the specific decision which is supposedly undertaken and the features of the environment. Therefore the ex ante representation of the choice problem reflects two components: (a) a *forecasting* of the future state of the environment and (b) a *model* which maps each alternative to specific consequences in that future state. The optimisation of the choice adds to all this a third component, consisting in (c) a *logical algorithm*, able to select the best one among a set of available alternatives. Each of the three components is very fragile from a cognitive point of view. Too fragile, indeed, to allow to consider the application of some optimisation algorithm to a choice set as some objective form of rational decision[3].

I shall explore, in the present work, only some particular aspects, though crucial ones, of this fragility; the key of the exploration will be based on the comparison between ex ante and ex post concepts of evaluation. For this purpose the evaluation algorithms, often complicated, will be reduced to their essential constituent elements, and shown to depend always, at least in one of their chain-links, on a purely mechanical extrapolation of past events; that is, on the correlation of the values of relevant variables with a *series of pure numbers* representing the flowing of time. This implies that any evaluation act, no matter how elaborated, is contingent to a (non rational) stake about the stability of such a correlation, that is, of at least one element of dull systemic inertia. Such correlation is widely acceptable, though always with caution, for most material events belonging to the physical sphere; it is not for most of evolutionary systems, such as the social systems.

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ganised; actually, on what decision theorists call "objectives function" (OF). They limit themselves to define the shape of the individual preferences. Decision theorists, instead, focus their attention on the modelling of the decision environment in terms of technical relations and constraints, and are obliged, in order to solve the decision problems, to specify the OF in each single application.

[2] Montesano (2005) clarifies the confusions arising from the different meanings that are attributed to the term "rational" in economics.

[3] In the present work I shall not make reference to the kind of fragilities that affect choices and judgements considered by Kahneman (2002) and Tversky & Kahneman (in plenty of works, but see in particular 1981, 1983, 1986, 1989), although the fragilities considered by them are continuously spilling over. I shall refer, rather, to a particular subset of what Stanovich & West (2000) call "system 2" processes (those based on reasoning and opposed to "system 1" processes, which are those based on intuition). The subset I refer to occurs when structured reasoning about choices is performed by high level teams with the best available differentiated/complementary competences.

This result induces to enquire about why evaluation based on optimisation has become so popular in our societies and among scientific communities and to reflect on whether and how it contributes or may contribute to the improvement of choices. This, however, should be done starting from a quite hard type of consciousness: the effects of the decisions depend on the features of the environment. The environment however, on its turn, is strongly shaped by the decisions undertaken by the single agents that belong to it, by their content, quality and degree of homogeneity, or by the distribution of different types of decisions. In such an uncertain, mobile and interdependent environment, to *define*, from a purely logical viewpoint, a *"correct" decision is an impossible task*, since its success depends not only on the inner properties of the variables which are under the control of the decision-maker, but also on the possibly changing shape of the environment as the result of the simultaneous and of the sequential decisions of other agents[4]. The very same action can succeed or fail as a consequence of the decisions of other agents[5]. An incidental corollary of the above proposition is that whenever I shall talk of "better", or of "improvements", this ought to be perceived as a probabilistic proposition, a "likelihood", *based on the value judgement* that "*more and/or more diffuse knowledge is on average better than less and/or less diffuse knowledge*".

The results of the enquiry are rich and someway contradictory, as it is summarised below.

1. The need to attach values to activities and to rely upon choice criteria that cannot be considered as objective, but are simply regarded to be so, is far from being useless. It stems from the inter-subjective need of establishing a fair consensus about the *soundness* of a proposed choice through the communication of summarised information concerning it. In this sense evaluation is today, like the new-born forms of accountancy were in the first half of last millennium, a way to communicate, by the means of an appropriate semantic format, among potential complementary partners in economic and social affairs; in brief, an institutional convention. Modern evaluation differs from old accountancy techniques in so far as it aggregates the relevant facts concerning the devised action according to weighting methods based on optimisation, instead of relying upon simple summation. Furthermore, since evaluation is a specific form of synthesis, it responds to the communication needs within and between the complex organisations that characterize our modern societies; eventually, since its elaboration is delegated to staffs, the functioning of the organisations is eased. It remains to be estimated whether the prevailing formats are adequate and, in particular, whether would it be better to give up a certain degree of synthesis and shift to more articulated arguments.

[4] This derives, in general, from what we know about game theory. I refer mainly, in any case, to Axelrod-type of games, where heterogeneous strategies compete each against the other ones, rather than to standard theory, where rationality and strategies are considered as homogeneous (Axelrod, 1984).

[5] This is put in clear evidence by the analysis of the performances of different strategies in different phases of Axelrod's tournament.

2. Since the evaluation requires the construction of a cognitive frame, it has a worth by itself as a *procedure*, which is partly independent of the particular criteria that are actually used. Evaluation has the nature of an institutional *construct* in a given moment of time, and that of *a process of* institutional *construction* in the course of time. Such construction is useful because it induces *processes* of decision making, and often *reiterations* of such processes, along which cognitive frames -and thus learning- are build-up.
   This applies to single organisations as well as to the systems resulting from their interactions.
   From the point of view of a single organisation, the need to perform an evaluation exercise compels the decision-maker to *put order into its imagination*. From a systemic point of view the evaluation creates a set of conventional rules whose function is that of conferring order, coordination and stability to the socio-economic system, through the establishment and the diffusion of routines, habits and beliefs.
3. While the need to evaluate may be originated by difficulties or by external constraints, as when a firm wants to share its property or obtain a loan, usually the performing of an evaluation exercise ends up in improving the *thoughtfulness* of the decision-making process. This is because, though a specific *evaluation criterion* may be deceptive, the fact of attempting to evaluate – that is, *the evaluation as a procedure* – is likely to improve someway the quality of the choice process. This amounts to say that it is not the evaluation in itself which is a good thing, but the fact that it compels people to *conjecture about possible futures*.
   This obliges them to gather, organise and exploit information, to select, develop or adapt causal models, to lay down the sequence of possible events that are likely to ensue certain decisions, to single out crucial conditional events and attribute probabilities to their occurrence. These cognitive tasks are shown better by statistical decisions analysis (SDA) than by standard economic analyses. At the same time it is easier to see the elements of weakness of any type of decision frameworks based on optimisation by using SDA as a descriptive benchmark.
   In all approaches based on optimisation too much attention is devoted to algorithms and criteria, too little to the appropriateness of the problem modelling. It would be likely better to consider the *choice criteria* in strict connection with the patterns of the environment specificities; that is, together with *the study of the subjects* who undertake relevant decisions and of their institutional links on the one side, and, on the other one, with *the investigation about the specific domain of problems* that the choice process is set up to solving[6]. The "art of conjecturing" about alternative futures, in other words, will be likely improved by considering together the *criteria*, the *in-*

[6] In many senses this proposition is opposite to the main implications of Paretian welfare economics. According to such an approach, the optimality features of a system made of perfect markets are sufficient to propose their equilibrium conditions as a rule-for-action to be pursued, independently of any further specific investigation.

volved subjects, the *problem domain* and *the models* which organise these pieces of knowledge, because all of them must enter appropriately in the structuring of the choice set and, at the meanwhile, because the weakness of a cognitive system always depends on the degree of weakness of its weakest component[7].

4. The likely improvements do not depend only on the fertility of conjecturing, but also on how the communication about it is conveyed, on the identity of the agents, on how the distribution of information and decisions affect the environment. The economic evaluations tend to summarise exceedingly the transmission of relevant information about the process, since they typically end up in a ranking of the available alternatives, with "numbers" associated to each of them. In such a way they tend to limit the cognitive participation to the process of choice (above all by hiding the role of the assumptions which are embodied in the used models, and often disguised behind their technicalities) and to hinder the control concerning its soundness[8].
   As for the identity of the agents, it should be acknowledged that the information is usually accepted in a weighted way: the information provided by those who have more reputation is more worthy, independently of the relevance of reputation for the evaluation of the proposed initiative. On the other hand, the success of inertial forecasting depends on the stability of the environment. The diffusion of decisions based on inertial forecasting, on its turn, reverberates on the stability of the environment. On the whole these features, together with other factors of distortion, risk to hamper innovative choices.

The conclusion of my survey of issues and problems concerning social decision making is that a honest optimisation exercise is seldom feasible when dealing with choices concerning human social interactions. This notwithstanding optimisation remains a very important intellectual benchmark, dense of suggestions about where to address our conjectures. This is, may be, because it is the only decision framework that is wholly "closed"; in other words we know perfectly the components that we should "know for sure" in order to perform the optimisation.

The problem is that we do not have such a firm knowledge. We should therefore address our efforts to deepening our knowledge where it is more shaky and doubtful, and we should be advantaged, in doing so, by the fact of knowing which is the more fragile component. But we should also, for this purpose, be courageous enough to accept our degree of ignorance and do our best to react, instead of cheating ourselves with comfortable assumptions.

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[7] This statement is analogous to Morgenstern's propositions about approximation. When this principle of caution is overlooked, many damages and waste are likely to be produced. Think for examples at the evil induced by actions chosen on the basis of perfectly well done economic cost-benefit analyses which did not pay enough attention to the anthropological features of the environment where the projects had to be realised.

[8] This applies above all to economic cost benefit analysis. It applies in a much lesser degree to serious applied efforts in multi-criteria analyses. Here the need to explore how different stake-holders relate to the choices elicits pertinent information gathering and allows more informed framing of the attitudes and values involved in the final choice.

"Thoughtful conjecturing" about possible and probable futures appears to me as a better, though extremely uncomfortable analytical guidance[9]. The reason why it is uncomfortable is clear: it is an open framework, resembling more to an "art" than to a scientific approach, more to an un-bordered path, that only strong and self-confident investigators can walk through, than to a well-closed labyrinth, getting out of which is only matter of method and perseverance.

This section summarises well the essence of what I think. The remaining part of the article argues my statements, or around them, not necessarily in the same order.

2 Naive Evaluations and Forecasting

A firm is considered to be well performing, according to most people involved in economic activities, if it has shown, in the past, to have on average a good profitability, as measured by indicators such as the cash-flow, the rate of profit, etc., although, as we will see, this way of evaluating may be regarded as rather naive. The implied goal is unique and has someway to do with profit. In the case of a single firm the performance P is usually evaluated according to different measures, or formats, each of them being a different function (a different expression), $P_\# = P_1, P_2, P_3, \ldots$, of the same set of dated variables, expressing the results (R) of the action as some function of the ratio between inflows from sales and outflows for producing what is sold:

$$P_\# = p_\#(R) \tag{1}$$

where $P_\#$ indicates the value of the performance of the firm according to the specific algorithm "#" and $p_\#$ is the corresponding symbol of function (more broadly, of "relation"). The important issue to point out is that the different algorithms apply to the same set of data, although any algorithm will usually weight differently each of its elements; that is, *it will attribute different meanings and importance to the same pieces of information*.

To this ex post evaluation criterion corresponds an ex ante one. Such a criterion has exactly the same shape of (1), but instead of the set of variables R, it is function of the expected values of R, $E(R)$:

$$EP_\# = p'_\#[E(R)] \tag{2}$$

where EP is the indicator of expected performance (of the firm or of one of its investments or activities); $p'_\#$ is any of the specific algorithms according to which the evaluation is made. The p' of (2) correspond to the p of (1), except that in (2) the logically corresponding functions are reformulated as different algorithms in order to apply to future variables[10] . $E(R)$ is the set of variables R such as they are expected in the future.

[9] I use here the term "analytical" in the same sense philosophers use it.

[10] For example because it takes the form of a present value computation.

The central issue is: how is the set E(R) obtained? In this approach the value of variables R for the future, R_{t+x} (where t is the time corresponding to the moment of the decision and $x = 1, 2, \ldots$), are determined as an extrapolation of the past values of R, R_{t-x}. The statistical algorithms used for the extrapolation may be very different. The way which is considered the more naive (but which, once again, is the more diffused in practice), is the following:

$$E(R_{t+x}) = f^* \left(R_{t-x}, S^*\right) \tag{3}$$

where f^* stays for different statistical algorithms, ranging from a simple average to the application of the linear regression parameters connecting R_{t-x} and the numbers $(t - x)$. S^* is a stochastic factor of deviation, whose positive or negative values are assumed to be sampled by a known distribution (whose average is usually assumed to be equal to zero); in particular f^* is the statistical operator according to which past and future variables are correlated.

In less naive cases, the estimate has the following shape:

$$R_{t-x} = r^* \left(A_{t-x}, S^*\right) \tag{4a}$$

$$E\left(R_{t+x}\right) = r'^* \left[E\left(A_{t+x}\right), S^*\right] \tag{4b}$$

$$E\left(A_{t+x}\right) = f^* \left(A_{t-x}, S^*\right) \tag{4c}$$

where A is a set of dated variables belonging to the environment which are correlated with R, r^* is any of the correlation algorithms in use, r'^* are the algorithms which imply the use of the parameters estimated with the corresponding algorithm r^*. Usually variables A are chosen in such a way as the estimate of their future values is considered as more reliable than that of the variables R. The fact of establishing a correlation among different (more or less simultaneous) variables, as in (4a), is considered as implying a better understanding of real phenomena (since the observation of correlations is usually enhanced by the specification of hypotheses about causal links).

This notwithstanding, the method (4) does not differ substantially from method (3) for forecasting purposes, so long as a moment of pure time extrapolation of past trends – here (4c) – is unavoidable, as it is easy to observe once one acknowledges that both, (3) and (4c), belong to the class of relationships summarised by the following expression:

$$F\left(Z_t\right) = H\left(t, S^*\right); \quad t = 1, 2, \ldots \tag{5}$$

where Z_t is any set of dated variables, F their possible transformations, and H a symbol of function. What (5) clearly shows is that the variable Z_t are put into correlation with the pure series of numbers t[11].

[11] There are cases in which this does not apply. Suppose that $R_{t-x} = r(A_{t-(x+y)}, S)$. Given the lag y, the actual observation of A in t allows a forecast for R in $(t + y)$ which does not depend on the correlation with pure numbers reflected by (5). The point is that this case is not of a great practical importance, because of three reasons: 1) the evaluation needs to be extended over a relatively long time span and this is limited by the magnitude of y; 2) the higher is y, the more the time series for estimating the parameters of (4a) have to be long, and good long time series are seldom available; 3) the higher is y, the higher is the probability that structural changes occur.

The reliability of the approaches summarized above is therefore subject to a fundamental assumption: that the identified parameters of correlation remain stable, which amounts to saying that the underlying structure of links between real phenomena will be unvarying through the relevant time span[12]. Its operational validity does not depend so much on the acceptability of its theoretical foundations, but on its success in forecasting which, on its turn, depends on the features of the environment. If the latter is stable, the forecasting is good, and its use is reinforced, contributing to stability.

When we come to innovative decisions, therefore, the above approaches do not apply by definition, since the aim of an innovative decision is that on inducing a structural rupture with respect to the given environment, with the likely effect of changing its features[13]. In such a case, that may be conveniently called as of *fundamental uncertainty*, the instruments of classical statistical analysis do not appear as being very useful, at least in themselves, and one cannot avoid facing the real nature of choices which have impacts upon, or depend on, future events: that of *conjecturing about possible futures*. Let us deepen, now, the different aspects of what I said.

3 Forecasting and Opportunity Values

A non-naive evaluation criterion, ex ante and ex post, must be based on the concept of opportunity cost (more in general opportunity values), that may be regarded as a case of counterfactual "thought experiments"[14]. Let us consider the ex post case first. In such a case we can put:

$$G_{\#} = g_{\#}\,(R_{\mathrm{Y}} - R_{\mathrm{N}}) \tag{6}$$

where $G_{\#}$ and $g_{\#}$ have a meaning corresponding to that of $P_{\#}$ and $p_{\#}$, R_{Y} is the set of ex post observed variables, given the choices made in the past by the decision-maker (and therefore R_{Y} amounts to R of previous formulas), while R_{N} is an estimate of the values that the same set of variables would have taken if other possible choices had been made instead of the one actually made. In other words the problem is to evaluate a choice *comparing the situation deriving from it with the virtual situation that would have occurred as a consequence of other possible alternative choices*. What is compared is the situation with, or without the choice; if the choice has been made (the subscript Y stays for "yes") or if it has not (N) been made[15].

[12] This also explains why we can trace back these ways of performing the evaluation to the evaluation of risk based on the extrapolation to the future of the frequency distribution actually observed for past events.

[13] Amendola and Bruno (1990).

[14] Mach, E., 1905, In particular "On Thought Experiments", pp. 134–147 in Mach, E., *Knowledge and Error: Sketches on the Psychology of Enquiry*, D. Reidel Publishing Co., (Dordrecht), 1976.

[15] Most recent benefit cost analyses actually consist of the comparison between two states of the world, one *with*, and one *without*, the adoption of the proposed project.

Formula (6) above does not convey the same information conveyed by (1). It has a non negative value if the choice has been good, and a negative one for a bad choice. Although this way of structuring the evaluation appears to be logically more correct than that conveyed by (1), the value G would be hardly understandable by common decision-makers, since it does not correspond to the prevailing culture. In fact, the evaluation according to (6) may be negative, despite the fact that the evaluation made according to (1) might be positive and very high; this would simply imply that an even more profitable option could have (but was not) chosen.

For such an ex-post evaluation a certain amount of conjecturing is needed in order to estimate R_N. The difficulty arises because, although historical facts are known, a part of them are strictly dependent on the fact that a certain choice has been made in a certain moment. Of the choices and events occurred since that moment, part of them would have occurred anyway, while other ones strictly depend on, and are consequential to, the first one. The analyst should thus face the not easy task of separating the two sets according to plausible analytical arguments.

When we turn to the case of ex ante evaluation, we face the same problem of conjecturing about what would happen if a given project *is not* adopted – but now without the help of historical information – plus the problem, that does not exist in the ex post perspective, of forecasting what might happen *if* the project is adopted. A proper expression of the problem of ex ante evaluation is the following one:

$$EG_{\#} = g'_{\#}\left[E\left(R_\mathrm{Y}\right) - E\left(R_\mathrm{N}\right), S^*\right] \qquad (7)$$

where the meaning of the variables should be clear by now.

The task of double forecasting outlined above not only poses the same problems that we considered in the previous section, but raises new problems concerning the way of conceptualising the conditional course of future events[16]. The estimate of $E(R_\mathrm{Y})$ implies an estimate of the kind captured by Eq. (5) above. Expression (7), however, raises further problems. While looking at the problem from an ex post perspective we have the information concerning the historical sequence of choice-options associated to the time path generating (R_Y), and these may be used as a benchmark for conjecturing about R_N , from an ex ante perspective the reconstruction by conjecture of the sequence of choice options associated with the adopting or not adopting the project would be not only really difficult, but such as to oblige the analyst to imagine and consider too many alternatives.

Thus, when the opportunity values approach is adopted, in order to avoid the combinatorial explosion of alternatives to be considered, *the conjectures about possible futures are usually truncated*. This is extremely evident when the techniques of scenario analyses are adopted, but occurs also with other techniques. *Truncations occur in every decision process; they are unavoidable*[17]. Decision-makers and

[16] In some sense the difference could be depicted by saying that the ex post evaluation resembles to a semifactual thought experiment, which can use plenty of inductive information, while the ex ante one resembles to a real counterfactual thought experiment, which can rely only upon logical conjecturing.

[17] As Simon puts it (e.g. Simon, 1955), no chess player would, or could, explore the whole range of possible combinations of sequences of moves.

their supporting analysts tend to reduce complexity by eliminating the exploration of events, or sequences of events, which are considered, *on the basis of intuition and experience*, likely to be less relevant and/or less likely to occur. However, the fact of using truncations along the investigative efforts is a further logical element which rules out the possibility that evaluation might be considered as an objective standard.

4 The Case of Multiple Goals

When we come to public decision-making and to the evaluation for public activities, the problem appears more complex. This higher complexity is partly apparent, and depends on the fact that the actual complexity of the decision-making processes in private organizations is usually underestimated and hidden by the someway artificial reduction of the goals of the firm to profit maximization. Expression (1) reflects the fact that a single objective is attributed to the firm. On the contrary, to the public decision-maker many goals are attributed. The value of a public action, or activity, will thus depend on how it contributes to the achievement of the many different goals attributed to the decision-maker. The value-meaning of the activity will thus depend on the results, expressed as the contribution that such results have in terms of each of the specified objectives.

If we agree to call R the results, as we did for the single firm, the evaluation function will depend on the "meaning", v, of such set R, in terms of each of the $i = 1, 2, \ldots, n$ specified objectives: $v_1(R), v_2(R), \ldots, v_n(R) = v_i(R)$. Therefore, we will have

$$W_{\#} = w_{\#}\,[v_i(R)] \tag{8}$$

where $W_{\#}$ and $w_{\#}$ have basically the same meaning of $P_{\#}$ and $p_{\#}$ of (1). In particular $w_{\#}$ implies that different evaluation algorithms are used by different analysts and/or environments.

The ex post evaluation function (8) differs from that (1) on many grounds. In a certain sense $w_{\#}$ reflects different ways (algorithms) for mapping the different value-meanings $v_i(R)$ in the synthetic measure $W_{\#}$, as it happened for $p_{\#}$ in (1) above. However, there is something more to it. In fact, $v_i(R)$ reflects the value-meaning of the result *for each of the objectives pursued* by the policy maker. But in order to obtain a *single value*, the different value-meanings have to be somehow added to each other since the different objectives are non homogeneous and their attainment is measured according to different arbitrary scales; in order to perform the sum, the single values v_i have therefore to be previously weighted. In order to do so, the worth of a given objective for the policy-maker in terms of all of the other objectives must be known in advance. In other words, both the n objectives and the $(n - 1)$ relative trade-offs among them (the whole objective function OF) have to be known in advance.

It should be noted that the values of such relative weights, or trade-offs, have no reason to be constant, neither with respect to time nor with respect to the absolute values of R and $v_i(R)$. Quite on the contrary, the weights will be likely to vary according to the absolute size of the degree of attainment of different objectives, and through time as a consequence of learning and of changing values[18].

Despite the fact that in the context of various disciplines dealing with the optimization of choices (such as linear programming, operation research, multi-criteria analyses, etc.) it is usually assumed that the decision- maker exhaustively knows, and therefore is able to communicate his/her complete OF, practical experience suggests a contrary evidence. Decision-makers are either not able to communicate their preferences in such a way as to allow reliable operative calculations, or they have distorted intuitions about them, and/or actually build up and structure such preferences along a process of learning activated by their interactions with the analysts. Concrete experience, in other words, suggests that the problematical aspects of the choice lay in the process of construction of a proper frame for involving the participating subjects; it is in the context generated by such a process that the subjects verify their understanding and feeling about the situation and what ought to be done, gather information and organise it, learn about the options and about their own preferences, transforming intuitive concerns into objectives specified in a way which is suited to be used operationally[19].

If we now shift to consider, for the decision-maker with multiple objectives, the ex ante evaluation problem, it is easy to see that everything we said about the conjectural nature of ex ante evaluation in the case of private companies applies here as well. In fact, if we suppose a relative stability of the value-meanings, in terms of each of the objectives, of the set R, we can put:

$$EW_{\#} = w'_{\#}\,[v_i\{E(R)\}] \tag{9}$$

or

$$EW_{\#} = w'_{\#}\,[v_i\,\{E\,(R_{\mathrm{Y}})\} - v_i\,\{E\,(R_{\mathrm{N}})\}] \tag{10}$$

according to whether a naive, or instead an opportunity cost, concept of evaluation, is used. The values of $E(R)$ have to be estimated according to one of the several methods considered earlier. Such estimates have to rely – at least in one of the

[18] Bruno (1986) argues that values are not independent of the evolution of technical information about the ways to attain the goals. See also Bruno (1984). In multi-criteria analyses the trade-offs are assumed to be linear, which means that they are assumed to be unvarying with the degrees of attainment of any single goal.

[19] Such a context for decision is, again, consistent with the hypothesis of bounded rationality as defined by Simon. Once the situation has been outlined in this way, most of the differences between the case of a private firm and that of a public decision-maker tend to fade away. The observation of the real behaviour of private companies, above all of large companies, tends to demonstrate that it is extremely hard to maintain that they have as their unique operational goal that of profit-standards. Modern companies, in fact, are observed as having multiple concerns, profitability being only one of them. From this point of view, at least, they appear to be more similar to public decision-makers than it is usually held.

steps – on a relationship such as (5) above, and thus meet the very same criticisms that we presented earlier.

5 Multiple Goals and the Flaw of Cost Benefit Economists

In the case of multiple goals there is a further problem, which has historically led to a heavy distortion in which the community of *economic* analysts has incurred.

Any intentional action may be regarded as producing (more properly "implying") a set of results R which is made by two distinct sets: on the one side there are results (subset R_v) that can be directly mapped onto goals (they are real "objects of desire" in themselves), on the other one there are "things" (a subset R_s) that are not important in themselves but only because their availability is instrumental to the attainment of the "objects of desire".

The evaluation of the elements of the subset of R_s within the logical frame of optimisation analysis is meaningful only in correspondence to the optimal solution of the underlying choice set (within this frame the instrumental variables are called resources and their values "shadow" values, or prices). This is perfectly sensible from a purely logical point of view, as it is the correlated proposition that the resources that are not saturated in the optimal solution have a zero shadow value while the saturated ones have a value which represents the contribution that a marginal increase in their availability would induce in the attainment of the objectives, measured in terms of a numeraire objective[20].

These concepts, however, have resulted hard to be digested not only in the everyday social life, but, surprisingly enough, by the majority of economic evaluators' communities and institutions, which are presumed to have a much more sophisticated ability. The confusion arises because of the un-resolved inconsistency between two different OF that the economic cost benefit analysts want to use together.

The economists believe that correct values are provided by the prices which clear perfectly competitive markets (ex general equilibrium theory, GET); they are considered as correct because they correspond to a Pareto optimum (ex the first theorem of welfare economics)[21]. Even abstracting from the possible criticisms of the Paretian way of aggregating individual utilities and from the severe limiting assumptions under which the GET properties hold, such equilibrium corresponds to the optimisation of individual preferences (with respect to market goods, as I shall explain later on). However this does not imply, if not apparently, an OF with multiple goals. The

[20] The shadow prices would not make sense in relation to non-optimal states, where an increase in the attainment of goals might be obtained by a simple rearrangement of the very same resources.

[21] According to Pareto the utilities of different individuals are not comparable. Therefore the only way of defining *scientifically* a better social alternative is to see if in such an alternative there is at least one individual who is better off and no one worse off. A situation where there is no Pareto-better alternative is a social optimum. The Pareto improvements require consensus among interacting partners.

market equilibrium amounts in fact to the maximisation of the total system's production of consumption goods, evaluated at their equilibrium prices. In particular it has been very precociously argued in the economists' community that perfect markets generate the same outcome that would be reached by a central planner having as its OF the maximisation of national product, that is, a single goal[22].

The cost benefit analysts, on the other hand, felt the need to add to this single objective also other types of objectives of social or environmental nature, overlooking the fact that any change introduced in the OF implies a possibly different optimisation outcome and possibly different systems of equilibrium relative values, for both, trade-offs between objectives (equivalent to relative prices of only consumption goods in the GET framework) and relative shadow prices for resources (equivalent to relative prices for inputs in the GET framework).

This error resembles to a Pandora pot; once it has been committed, no obstacle remains, and so one can find in the Operational Statements of the World Bank indications about two different and coexistent sets of shadow prices: the social and the economic ones!

6 Statistical Decision Analysis (SDA), the Framing of Conjectures and Inter-subjective Communication

We have only scattered hypotheses about how new models and ideas derive from old ones. For the moment, therefore, it is safer to attribute the mental activity of conjecturing to the domain of the art[23] and to consider it as belonging to the sphere of the imagination. Being an art, its patterns and quality strictly depend on, and interact with, a contingent system of referees[24]. The reference system is made of codes, ranging from techniques to aesthetical patterns. The codes bind the artist, but also help him/her in framing his/her imagination, in giving substance to it, in rendering its results communicable to the environment. At the same time it is clear that the innovative variances in the way of elaborating the codes may cause, if they gather enough consensus, an evolution of the accepted codes, i.e., of the reference system.

In a similar fashion the way the conjectures about the effects of an action are developed in a project is thus unlikely to be free, but likely to be framed by a system of rules and procedures[25] which are contingent to the environment, but at the same time likely to evolve, in the same way and according to the same complex interactions, encompassing invention and learning, considered for art codes.

The evaluation that we considered at length earlier is one of the possible ways to frame a choice-set. A much more explicit way of doing it is that suggested by the SDA, which I regard as one of the great achievements of past century, through

[22] Barone (1908); Lange (1936, 1937); Samuelson (1948).

[23] Feyerabend (1984).

[24] Perelman-Olbrechts Tyteca (1958); Kahneman (2002); Berger (1999).

[25] Tverski et al. (1989).

the so called "decision tree"[26]. SDA offers a framework for organising the way we conceive our possible futures, made in such a way as to exploit some logical hints of probability theory[27].

The value of each alternative depends simultaneously on the pay-offs associated to any conceived result and on the structure of subjective probabilities attributed to uncertain events. In this framework, the double "relativity" of the evaluation process is extremely evident. Although nothing forestalls that the pay-offs be given in money terms, their actual use is that of a consistent ordering of any kind of "objects", that admits transformations as long as the transformation criterion maintains its consistency through the pay-off matrix. The evaluation is thus contingent – that is, relative to – the specific object, or collection of objects, that the decision-maker has decided is important and worthy to be considered. Furthermore (and this is the second "relativity"), the attribution of probabilities to the uncertain events that condition the outcome is treated as dependent on the subjective sensitivity and experience of the decision-maker. However this experience may change, in the course of choice-framing, on the basis of impulses endogenous to this process, and this is something completely new: starting from a given structure of attributed probabilities, a targeted acquisition of further information is found to redistribute the probabilities of the events in a non-ambiguous direction in such a way as to improve the efficiency of the final choice. The very same process of decision framing performs as an engine of search.

These features unveil the essence of the process of conjecturing about the future more clearly than in the case of evaluation based on economic optimisation. As we saw, the latter one usually epitomises its evaluative function by reducing information to a set of synthetic economic magnitudes, offered as if they were the only relevant "pieces of information" about the future, and not simple "guesses" about it. This is because the future is usually forecasted in a deterministic way, and not represented as an articulated conjecturing about possible states of the world. Furthermore, in the classical evaluation analysis, the factual magnitudes, or "events", are usually not separated by their value implications. The decision tree, instead, separates the events and their value-attributes, and distinguishes, among the latter ones, between probabilities and utilities (or value equivalents). The role of the decision tree may be regarded thus as that of making things clear to the decision-maker and to shape them in such a way as to render it possible to communicate them to third parties. Finally, the SDA is able to suggest the direction and the opportunity of further enquiries, though they cost and take time, and allows a better potential control upon each of the

[26] The origin of such a cognitive tool-box may be traced back to von Neumann-Morgenstern (1944). But see also, for a simple explanation, Raiffa (1968).

[27] The first step is that of identifying the consequences of given "alternative" actions, should certain events occur with certainty. A preference ordering of such consequences has then to be stipulated (the so called pay-off matrix). The events, however, are usually uncertain; the next step is therefore that of attributing a subjective probability to such events. In this phase of the exercise, all of the existing information (and thus also that which is made available by classical statistical analysis) may be utilised.

steps of the decision process, either made by the same decision-maker or performed by third parties.

7 Probabilities of Error in Models' Selection

SDA too, unfortunately, hides an essential part of the cognitive chain which is at the heart of conjecturing about alternative states of the world; namely, *how do we design the structure of the decision tree, and why do we do it in a certain way.* We can attach certain probabilities to certain events and certain pay-offs to other ones *only because* we have *previously* established someway that exactly *those events*, and exactly *that particular set of connections* among them, are relevant with respect to our particular problems. In brief, *the meaning of probabilities and pay-offs is contingent to the fact of having selected a specific "model"*, that is, a particular structured system of hypotheses, for representing the problem[28].

The problematical aspects associated to the choice of a particular model (why the selected model is regarded as being "good" or "more appropriate") are usually ignored or overlooked. What is usually hidden under the straightforward choice of a particular model is that the decision maker or his/her supporting analysts rule out the probability that their particular selected model may be wrong or anyway inappropriate, while they exclude that other alternative models might have probabilities substantially larger than zero to be appropriate[29]. This is a wrong attitude.

Given a problem, there are usually several models which are possible candidates for dealing with it (in my definition the "models" are defined as the lenses through which we observe the real world[30]). According to modern epistemology, none of them may be stated as correct, or true, though each of them might be considered as being differently corroborated. The choice of a model for dealing with a problem should thus be regarded in itself as a decision under conditions of uncertainty.

In a more complete analytical treatment, *a third relativity* should thus be explicitly considered, concerning the selection of the cognitive model which carries us to outline in a certain way the choice-nodes and the uncertain events which condition the future. More than one alternative models can be selected, attributing to each of them a probability of being appropriate.

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[28] See Bruno (1984, 1986).

[29] See the last chapters of Hacking (1975), above all the two chapters pivoting around Jacques Bernoulli's *Ars conjectandi* and the correspondence between Bernoulli and Leibniz, and Hacking's positioning of Humes' contribution to induction. Kahneman & Frederick (2002) offer plenty of evidence about how and why relevant knowledge is eliminated, not only when intuition but also when structured reasoning are involved; in other words when people shift from system 1 to system 2 cognitive processes.

[30] Notice that the way the term "model" is used here highly differs from the way it is used by most statisticians and many economists. The difference may be gathered considering that, when my concept of model is used, the emergence of new information concerning the events may improve the estimates of what is forecasted by the model, but does not, by itself and necessarily, improve the model. The model may change (and thus also possibly "improve") only in so far as new information leads to a reshaping of the set of hypotheses and/or of their structuring connections.

The information concern the shapes of the real world, and do not affect how the lenses are made. It might happen, though, that the experience in observing new shapes of the real world may lead occasionally to design new and apparently better performing lenses. The epistemological problems involved in the construction of the decision tree (as well as of any choice-set) are thus immense, though usually overlooked: actions have to be selected from an open cognitive inventory space, their consequences have to be derived through the use (sometimes the creation) of one or more models drawn from an equally open space, the objectives and their metrics have to be defined unambiguously, etc.

8 Evaluation as a Rhetorical Device

If we consider the issue from the this vantage point, namely that of fundamental uncertainty, a further important element becomes clear. While the more traditional ex post evaluations have an informational background made of facts and of their observation, the dependence of ex ante evaluation on the system of hypotheses and conjectures about an uncertain future defined according to counterfactual reasoning confers to it an essentially *qualitative* nature.

Evaluation, in other words, is nothing else than a quantitative specification of certain conjectural aspects of the process of constructing and analysing a choice-set[31]. The *numbers* associated to an ex ante evaluation should be taken as saying that the preferred option lies in a range which is below or over certain critical values[32]; furthermore, it has such quantitative properties only because of the interplay between the arbitrary values attributed to the pay-offs and the equally arbitrary values attributed to the probabilities associated to uncertain events; events which are, in their turn, arbitrarily identified through the use of arbitrarily selected causal models. "Arbitrary", however, does not mean "stupid" or "meaningless". It simply implies that we are not able to establish a benchmark for objectivity, if not based on conventions. But conventions require sharing of beliefs, and/through communication[33].

This proposition puts, from an epistemic point of view, the socially relevant decisions stemming from technical procedures and (apparently) objective rational methods at par with those based on political procedures and stemming from political

[31] This is an aspect of the ambiguous nature of the concept of probability, which sometimes appears to account mainly as an epistemic attribute and in other occasions as a quantitative standard of likelihood (Hacking, 1975).

[32] It is not by chance that this reasoning reminds closely the argument of Simon (1955) about the thresholds for satisfycing; but it has also to do with the dual nature of probability, considered as a measure or as an indicator of degrees of belief.

[33] Feyerabend (1984); Kuhn (1970); Hacking (1975) argues that the original meaning of "probable" had little to do with induction, but meant "approvable". In the late middle-aged propositions were considered "approvable" as long as they were corroborated by one or more scholars endowed with a conventionally agreed degree of reputation. Probability then, as well as evaluation in my present argument, have thus the common feature of depending on social conventions.

processes. They both share a basically rhetorical nature[34], although differ in the way of inflecting it. Such differences have to do – among other elements – with the range of application, with the methods and semantic formats that are used, with the nature of involved agents and environments.

The political decisions stem from complex processes in the course of which what matters (at least where what we usually regard as democratic systems prevail) is, on the one side, the constitutional/institutional frame that binds the decisions in terms of procedures and constraints, and, on the other side, the "style" of the political debates and negotiations which affect the content of the decisions. On the whole this amounts to a very important but relatively vague system of boundaries for what can be decided and how, at least if one compares it with the set of methodologies and rules that bind "technical" decisions[35].

The rhetorical component emerges in a completely different way in the two types of decision processes. In the political processes such component is relatively evident, since the goal of the involved parties is usually more that of persuading the majority of other ones that its pre-conceived solutions should be adopted than to search the best solution through dialogue. At the same time and quite consistently the existence of conflicts, and of the interests and the ideologies that feed them, are not only evident, but indeed central. In technical decisions, instead, the rhetorical component is hidden, while conflict is variously expunged. It is interesting to deepen how, since there are important differences between different technical approaches.

In most multi-criteria analyses (and more in general in operation research) the objective functions (OF) are taken as exogenous and referred to a single and sovereign decision maker. Therefore the conflict aspects can be considered to be solved or settled in the upstream phases of the process of decision making, that is, before the moment in which the analysts are involved.

Occasional enquiries, made in many multi-criteria analyses, about what further stake-holders want may be regarded in most cases as a corroboration or a qualification of already sufficiently defined OF or, though more rarely, as investigations to be placed at the service of the decision maker in a phase in which he/she has still to make up his/her mind about his/her goals. Otherwise they are inconsistent. These considerations, on the whole, apply also to SDA's. The rhetorical component is completely embodied in the very same operation of hiding or understate the factors of weaknesses, from the point of view of rationality and objectivity, that have been analysed earlier, and the role that has on the one side the identification of the problem that the analysis is set up to solve at best and, on the other one, the role of the assumptions embodied in the whole process.

The latter considerations about the rhetorical component apply to economic evaluations too.

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[34] In the modern sense of Perelman-Olbrechts Tyteca (1958).

[35] This contributes to explain why "technical" decisions are more homogeneous across countries than political ones.

For what concern conflict, instead, the cheating apparatus is much more sophisticated and lies in the very same roots of the welfare implications of the General Equilibrium (GE) paradigm. It requires a deeper analysis, that can be only outlined here.

9 Why the Use of Market Values is Deceptive

In my view conflict is expunged by surreptitiously *eliminating either the objects of desire* that can be controversial, with the remarkable exception of the distribution of wealth (the individual endowments)[36], *or the subjects who evaluate in an opposite way the same "things"*.

The origin of the deception can be traced back to the fact that the founding fathers of the so called "marginalistic" (or neoclassical) revolution that originated the GE approach, namely scholars such as Walras, Jevons, Menger, Edgeworth, considered marketable commodities as being *the* arguments of the individual utility functions (namely, as the only considered "objects of desire"). Given such a domain of assumed useful "things" for all interacting individuals, and by assuming further that any of these things may at worst be simply "non useful" for some of them, it was ruled out or regarded as an exception the possibility that a very same event might be desired by certain individuals and, at the same time, disliked by other ones. Once given for granted that all individuals share a positive or null appreciation for any produced thing, but not a negative one, the same attitude has been extended to the case of public goods considered by Lindahl and Samuelson, that were regarded (always) as "goods", that is, goods that, once existing, positively affected everybody.

Notice that not only the first mentioned scholars, but, later on, also the scholars of the following generation, such as Pareto and Pigou, were perfectly conscious that they were someway forcing the utilitarian premises, that implied that most human beings draw happiness from many more events than only by riches and commodities, such as beauty, freedom, affective relations, ethical behaviours, etc. . However, they were restricting their attention to commodities and riches only because they were attempting to construct an autonomous "science" for analysing economic events, but were perfectly conscious of such a distortion, that they attributed to the still immature stage of evolution of science. The limit case was Pigou, who went so far, in his *Economics of Welfare*, as to accepting for himself a statute of "non-scientist", or scientist in a week sense, in order to make normative propositions in the economic domain[37]. Unfortunately, this consciousness faded away later on, when most welfare economists on the one side adopted the Pareto principle for defining social

[36] That is the only case of conflict that is considered by the economists.

[37] He did so by assimilating himself more to Marconi, the inventor, than to Hertz, the *scientist* on whose ideas Marconi worked out his inventions. Pigou, indeed, spent his whole first chapter to discuss the distinction between "economic" welfare and welfare in general.

optima[38], but, on the other one, inconsistently adopted such a principle for normative purposes[39].

The end-result of such a chain of distortions is the first theorem of welfare: any equilibrium occurring as the result of interactions in perfectly competitive markets is a Pareto optimum. Actually this outcome is perfectly logical but only in the abstract and artificial environment deriving from the distortions considered above, since a market equilibrium is generated by the exhaustive exploitation of the existing agreement options in a word where such options are restricted to market interactions and where individuals optimise only in the domain of marketable commodities[40].

The point, however, is that such an environment does not resemble to any existing world, where cases of conflict, well beyond that of distributional justice, are likely to be frequent. These are the cases when individual actions or agreements between a subset of partners are disliked by other subjects (negative externalities).

Furthermore, there are important options of consensual agreements that can be captured only outside the market, mainly through political interactions; think for example to the collective pursuing of ethical or aesthetical goals shared by many but not necessarily all the individuals[41].

One can plainly hold that in any world where the latter cases, as well as cases of conflict, are relevant, the frontier of results generated by market interactions is systematically dominated, from the collective welfare point of view, by a Pareto-like frontier, generated by taking into account negative externalities (by restricting or regulating the market trades) and by systematically exploiting the non-market options of agreement, performed in such a way as to take into account (in some way to be specified) the desires of those who oppose them.

This implies that, outside the Arrow-Debreu axiomatic world, market values are far from constituting a benchmark for evaluation, as the wide majority of economists perseverate in holding, despite the unambiguous arguments of Arrow Debreu. Their position, to say it in even clearer words, implies that *the values which are generated by a subset of individuals who stipulate contracts having as their object a subset of the objects of desire are asserted as being the appropriate benchmark for evaluating everything for everybody, in the market or outside it*; a benchmark, in particular, that can and must be used as a neutral "technique" for deciding.

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[38] See footnote 21.

[39] Pareto was extremely clear in excluding any normative implication from his scientific considerations.

[40] This is extremely clear in the axiomatic treatment by Arrow-Debreu (1954), that ultimately demonstrated the existence of the equilibrium solution but, at the same time, illustrated the strict conditions under which such a solution exist.

[41] Should all the individuals share the goals we would be in the public goods case considered by Samuelson. In other cases we would fall in a further case of possible conflict, as it would be, for example, when all agree on a defensive armament but a part of the individuals oppose more armament than that which is strictly needed for defensive purposes, even in the case in which they have not to pay for it.

10 Asymmetric Information and the Reputation of the Decision-maker

Despite the limits put forward in the last two sections and the risks that will be illustrated in this and in the next one, the evaluation techniques are a way to push individuals, organisations and governments towards conjectural thinking and to set out more precise formats for communicating and ease the attainment of consensus among complementary agents.

The formal decision criteria adopted in each evaluation approach may be better or worse, may be more or less appropriate to the choice at stake, may improve; however *the fact of using some criterion*, that is, *the fact of adopting a procedure for controlling*, is likely to have effects, socially regarded as positive, on the decision making process, on the quality of its results and on the consensus upon the choice.

When labelling evaluation as a rhetorical device, I raised an issue which can concern also the inter-subjective transmission of the knowledge-content embodied in the evaluative conjectures. The need to transmit the information, both within an organisation and towards agents external to it, presupposes an initial state of asymmetric information among the involved agents and that the aim of communication is that of reducing (or often simply "moulding") the pre-existing patterns of asymmetry, as a condition to reach the threshold of consensus which is necessary to adopt a given proposed action. The acceptance of a proposed choice is always thus a question of shared *beliefs*.

Common experience suggests, however, that an action is rarely considered independently of the specific agent who is proposing it. In other words there is always an implicit issue concerning the trust towards the agent, or the agents, who take the ultimate responsibility for the conception and the implementation of the action. This statement is important since it stresses that there is a trade-off between the information concerning the object of the decision and that concerning the subject who proposes the choice and takes responsibility for it. Also the information about the subject, in any case, is a conjecture taking the form of an extrapolation: if subject z has shown to be fairly reliable and successful in the past, he/she can be safely regarded as likely to be equally performing in the future[42].

However, when dealing with the information concerning the object of the choice, what matters is the apparent robustness of the conjectures communicated through the project; that is, the structure and the width of the arguments of which it consists, the persuasiveness of the used models, the pertinence and relevance of the inductive evidence, etc. Instead, as far as the information about the proposing subject is used, what is evaluated is the degree of reliability of the subject. The likelihood of the acceptance of a project and of the support to it depends thus on the *curriculum* of his/her proponent, and this explains why new firms, or firms not having a *curriculum* of successful actions, have a disadvantage in obtaining trust and cooperation. *Coeteris paribus*, they must then show better projects, and, even so, they are likely

[42] This is again a logical flaw well explained in the tradition of framing (Kahneman, 2002).

to remain disadvantaged. *Curricula* holders, on the contrary, can afford to rely upon more intuitive and/or more reticent projects.

Such considerations have far reaching implications from a systemic point of view, since on the one hand many interesting innovative options risk to be overlooked only because their proponents do not have sufficient reputation, while, on the other one, the innovative options which run more the risk to be imitated tend to be more advantageously managed by firms which have more reputation, because of their larger margins for being reticent about the features of the option. Furthermore, since reputation is by definition a cumulative phenomenon, it is likely to be unevenly distributed among agents, without any guarantee that the distribution of reputation corresponds to the distribution of innovative abilities[43].

11 The Case of Innovative Decisions

Although procedures do not coincide with routines, often they are strictly associated to them or, more exactly, they are among the rules which structure a routine. Routines help in running smoothly organisational tasks, through the establishment of rules and mechanisms able to connect a system of situational patterns to an appropriate set of behavioural responses. Such rules and mechanisms, however, tend to acquire a momentum of their own, which may become a factor of resistance against the adoption of innovative decisions. This applies, in particular, to the prevailing evaluation procedures.

The habit to what I argued to be "pseudo-objective" criteria for evaluation, although contributing to the efficiency of private concerns in normal circumstances, has thus drawbacks as long as it plays against innovative decisions. As I noticed earlier, in fact, innovative decisions are not easy to be evaluated in terms of standard criteria; they are nothing else than "sound stakes"; they cannot but be based quite explicitly on structured conjectures. However, the people who, within the firm, deal with the budget, or the share-holders to whom it is demanded to risk their money, or the banks, often do not accept as convincing the proposals that cannot be evaluated according to standard routine criteria. This may create a difficult and unfavourable situation for innovative decisions. If an action is really innovative, nobody can actually tell which, for example, is the prospective rate of return on investing in the project concerning such an action, although many people pretend to[44].

Four factors matter in the success of an innovation. One of them – chance – is out of the grip of any innovative organisation; however such an organisation can estab-

[43] This may be regarded as a case of externalities that may produce bad allocation of resources, that should require public intervention. This might be done through the creation of a public agency which should consider innovative projects using a degree of perceptiveness and of sensitivity inversely correlated to the reputation of the applicants.

[44] The reason they pretend is obviously rhetorical: there is someone who wants to carry on the project, the validity and interest of which appears clear to them, but he/she must convince other parties, within or outside his/her organisation, that it is worthwhile to finance the project.

lish some, though never complete, control on the other three of them – learning[45], feasibility[46] and liquidity. The probability of success depends thus on the control that an innovative organisation, let's say a firm, is able to establish upon them. The ex ante "evaluation" of an innovative project, i.e. the conjectural elements that will cause its adoption or refusal, depends on the confidence that the firm itself, or its prospective partners (the shareholders, the lenders, the cooperating firms or even the state, when it acts promoting innovation), have upon the ability to exert such control. From the point of view of inter-subjective relationships, liquidity is the more important and the more binding of the factors considered above[47]. The performing of innovation usually requires an increase in the ratio between financial outflows and inflows. Such an increase in the rate of financial needs may be fed by internal and/or external sources. In the first case the rate of innovation cannot exceed certain values, which depend on the ratio between traditional activities, which provide the liquidity, and the innovative ones. In the second case, those who are requested to risk their money have to be convinced to share the stake, and here cultural and informational asymmetries are likely to constitute the limiting factors. In this framework the fact that those who sponsor the innovative choice pretend to know its rate of return

[45] There are different types of innovations: a firm may simply substitute the technology with which to produce the same commodities it was already producing, or decide to produce new goods and services with proved technologies, or finally to build up by its own an entirely new innovation. Each of these alternatives implies some form and some degree of learning. The process of learning and its complex patterns are more evident in the latter case.

According to Amendola-Gaffard (1988), a genuine innovation implies the choice to *take a path* whose end-result is only vaguely defined when the choice is initially made. The innovation takes an increasingly definite shape along the process of its construction, essentially regarded as a learning process. Labourers work to building something new, and in so doing they learn, changing themselves (their knowledge and abilities) while further defining the features of the new technology to come. Furthermore, since the quality structure of the labour input in any given period strictly depends on what labour has been set to do in the previous ones, the range of things which are possible from a given period onward is someway limited by the actions which have been undertaken in the previous periods.

[46] Here again the more extreme case of innovation may help in clarifying the problems involved by the undertaking of an innovative path. Amendola-Gaffard (1988) insist particularly on the problems of intertemporal complementarity that characterise the innovation considered as a sequential process (see also the previous footnote). Other contributions have stressed, instead, what we may call in general "horizontal" complementarities, i.e. the set of complementarities that the innovating firm has to set with its surrounding environment (other firms, the labour markets, public agencies and institutions, the clientele, etc.), and/or to exploit, in order to successfully develop the innovative fact (Kline-Rosenberg, 1986; Freeman, 1987; Lundvall, 1988; Silverberg-Dosi-Orsenigo, 1988; Torrisi, 1988; Mariotti, 1989; Teece, 1989; Vacca'-Zanfei, 1989; David, 1990; Nelson, 1990; Amendola-Bruno, 1990; David, 1991). Also the setting up of such complementarities has a dynamic sequential nature.

The first concern of the innovative agent should thus be that the envisaged process might be viable.

[47] Since the building of something new or additional requires that labour be devoted to its creation, this implies a lengthening of the average period which elapses from the beginning to the end of the production processes, and thus of the lag between the moment when labour begins to be paid and moment when the output may be sold. This corresponds to larger financial needs.

amounts to little more than adopting a sort of what in the military jargon is called a "counter-device".

12 The Importance of the Problematical Environment

I have already argued about the substantially qualitative nature of the variables constituting the object of conjecturing which is associated to the choice process. What is essential in the construction of a good project is, first of all, the identification of the possible consequences of the devised actions and of the events which condition them. Such a step is preliminary to any economic consideration, and requires the selection of models which are appropriate to the type of problems which are faced by the decision-maker. The choice requires thus a deep knowledge of the technical and/or social problems involved and of their context, the availability of good analytical tools and of reliable information specific to the environment. The same elements confer strength to the attribution of subjective probabilities to the uncertain events which condition the results.

The economic aspects may emerge in a later phase, when dealing with the economic results (more in general with the pay-off matrix). However it is quite clear that a correct identification of the pay-offs is somehow secondary with respect to the correct identification of the structure of the decision-tree (the outcomes and their conditioning events), i.e. the choice of an appropriate model or set of models.

Once again, reflecting upon innovative actions may contribute to the deepening of the cognitive problems involved. According to Amendola and Gaffard an innovative choice is one which gives start to a constructive sequence along which an innovation may take shape. What the initial decision opens is thus a path which not only has many bifurcations, but in which any further decision opens, or simply makes it possible to perceive, further bifurcations. Suppose to represent, in $t = 0$, the initial choice-set by the means of a decision-tree. Given a represented set of possible outcomes in $t + n$, some, or all of them may be regarded as a sketchy description, or as an intuitive summary, of further decision-trees, the contours of which will become more definite not only as the time elapses (as it is obvious), but *as a consequence of having undertaken, in previous periods, certain specific actions* (instead of other ones). In other words, the actions (the very fact of acting) contribute to reshape the decision-tree and enlarge the set of available information, doing it in a way which is contingent to the specific action actually undertaken.

On the other hand, this neither implies that sound conjectures could not be made, nor that it is impossible to improve the scope, the structure and the width of such conjectures, nor, finally, that such conjectures are useless. Quite on the contrary, there are wide options for utilising the increasingly powerful instruments that different scientific branches are elaborating. Models of numerical simulation are among them, since the setting up of innovation depends, for its success, on a set of intertemporal and inter-subjective complementarities, the setting up of which, too, has a dynamic sequential nature that can be captured in an easier way by numerical simulation[48]. The sequential nature of innovative processes implies that one has to perform certain activities in t if he/she wants to have certain results in $t + x$, where x is the time of construction of the relevant results. The environment, however, will be somehow modified in the time interval between t and $t + x$ by the indirect feed-backs generated either by the actions undertaken in $t - x$ or even by those undertaken in t. Furthermore, for the inter-subjective complementarities, the relevant decisions are not only those undertaken by the firm that begins and leads the innovative process, but also those undertaken by other agents acting in the environment, that can be either independent or, instead, such as to be affected by prior influences exerted by the former innovative agent. In this case there is a complex problem of time scheduling and of coordination (Richardson, 1960; Bruno-De Lellis, 1993).

The minimal condition for success is that the process be feasible at least for one of the possible sequences. On the other hand, the larger the set of likely feasible sequences and/or the looser the time-path conditions, the more promising the envisaged innovation would be. This proposition should be taken as having a likely normative value. In conditions of fundamental uncertainty and for actions whose implementation is characterised by a complex web of inter-subjective and inter-temporal complementarities, the tests conceivable on the basis of the above proposition would be likely to result of greater strategic importance, and anyway preliminary, with respect to more traditional evaluation tests[49].

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[48] It is to notice that numerical simulation, while today widely accepted in many scientific branches, and notably by the physicists, is substantially refused, as a theoretical instrument, by the economists.

[49] On the whole, this outline highlights the opportunity to use or adapt techniques able to couple probabilistic and reticular analyses (Nicolo', 1990), with the occasional support of hints derived by game theory (Petit, 1991) in order to consider the possible moves of competitors. In any case, since the dynamics of transition are quite tricky, numerical simulation should be used for elaborating sequentially-built scenarios.

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Part II
Agent Based Models

A New Agent-based Tool to Build Artificial Worlds

Pietro Terna

Abstract. We propose SLAPP, or Swarm-Like Agent Protocol in Python, as a simplified application of the original Swarm protocol, choosing Python as a simultaneously simple and complete object-oriented framework. With SLAPP we develop two test models in the Agent-Based Models (ABM) perspective, building both an artificial world related to an imaginary situation with stylized chameleons and an artificial world related to the actual important issue of interbank payment and liquidity.

1 A few Notes on Agents and Complexity

Following Ostrom [12], and to some extent, Gilbert and Terna [7], in social science, we traditionally build models as simplified representations of reality in two ways: (i) verbal argumentation and (ii) mathematical equations, typically with statistics and econometrics. The first way (i) is absolutely flexible and adaptable, as in the case of a historical book reporting an analysis of past events, but mere descriptions and discussion, by their nature, preclude tests and verifications of hypotheses. In contrast, the second way (ii) allows for computations and verifications, but suffers from severe limitations in flexibility and adaptation, especially with respect to how agents are expected to operate in the model and when accounting for their heterogeneity and interactions.

There is a third way to build models, (iii) computer simulation, mainly if agent-based. Computer simulation can combine the extreme flexibility of a computer code where we can create agents who act, make choices, and react to the choices of other agents and to modification of their environment – and its intrinsic computability. This allows us to use the descriptive capabilities of verbal argumentation and the

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ability to calculate the effects of different situations and hypotheses together. From this perspective, the computer program is a form of mathematics. In addition, we can generate time series from our models and analyze them employing statistics and econometrics.

However, reality is intrinsically agent-based, not equation-based (for a short, but illuminating discussion of this consideration, see Weinberg [18] in his review of Wolfram's book, A New Kind of Science). At first glance, this is a strong criticism. Why reproduce social structures in an agent-based way, following (iii), when science applies (ii) to describe, explain, and forecast reality, which is, per se, too complicated to be understood? The first reply is that we can, with agent-based models and simulation, produce artifacts of actual systems and "play" with them, i.e., showing consequences of perfectly known ex-ante hypotheses and agent behavioral designs and interactions; then we can apply statistics and econometrics to the outcomes of the simulation and compare the results with those obtained by applying the same tests to actual data. In this view, simulation models act as a sort of magnifying glass that may be used to better understand reality. Considering the analysis of agent-based simulation model as a source of knowledge, there is another "third way view" of these kinds of tools. Consider Axelrod and Tesfatsion [3]:

> Simulation in general, and ABM in particular, is a third way of doing science in addition to deduction and induction. Scientists use deduction to derive theorems from assumptions, and induction to find patterns in empirical data. Simulation, like deduction, starts with a set of explicit assumptions. But unlike deduction, simulation does not prove theorems with generality. Instead, simulation generates data suitable for analysis by induction. Nevertheless, unlike typical induction, the simulated data come from a rigorously specified set of assumptions regarding an actual or proposed system of interest rather than direct measurements of the real world. Consequently, simulation differs from standard deduction and induction in both its implementation and its goals. Simulation permits increased understanding of systems through controlled computational experiments.

The considerations above act in a way similar to abduction, or inference to the best explanation, where one chooses the hypotheses that, if true, give the best explanation of the actual evidence. Note that in the ABM perspective, the hypotheses are also related to the rule that determines the behavior of the agent.

The second reply is that, relying on Anderson [1], we know that complexity arises when agents or parts of a whole act and interact and the quantity of involved agent is relevant. Furthermore, following Villani [17, p. 51], "Complex systems are systems whose complete characterization involves more than one level of description." To manage complexity, one needs to build models of agents. As a stylized example, consider ants and an ant-hill: Two levels need to be studied simultaneously to understand the (emergent) dynamic of the ant-hill based on the (simple) behaviors of the ants.

We can also imagine building models based on multiple layers of agents[1], with the agents of each layer composing in a collective sense the more complicated agents of the upper stratum.

[1] Being each layer a *swarm*, which is also the name of the first standardized tool used to build this kind of models, i.e., Swarm, from Santa Fe Institute, in Minar et al. ([11]).

This interpretation of the agent-based paradigm, which is consistent with the way this kind of model is used in this work, corresponds to the "second use - partially soluble models: Artificial agents as complementary to mathematical theorizing" and to the "third use - models ostensibly intractable or provably insoluble: Agent computing as a substitute for analysis" considered in Axtell [4]. The Axtell's first use occurs "when models can be formulated and completely solved: Agent models as classical simulation."

The first use quoted above is mainly related to Monte Carlo simulations and the verification of numerical solutions to equation models. The second use relates to the cases of existing equilibria which can be incomputable, not attainable by bounded rational agents, known only for simple network configurations, or less interesting than transition phases, fluctuations, and extreme events. The third use is related to intractable models (my addendum to Axtell's considerations) when we believe that agents should be able to develop self-generated behavioral rules. This is the case here.

However, agent-based simulation models have severe weaknesses, primarily arising from:

(a) The difficulty of fully understand them without studying the program used to run the simulation;
(b) The necessity of carefully checking computer code to prevent generation of inaccurate results from mere coding errors. Epstein and Axtell [6] pointed out, in their seminal paper, that it is necessary to develop new ways to control software and avoid bugs. In addition, due to the object-oriented structure that is intrinsic to agent-based programs, it is also possible to create a class of internal agents charged with observing the behavior of the actual agents of the simulation and reporting anomalies. Anomalies can be interesting to analyze and do not necessarily always arise from errors, but it is necessary to carefully explore this possibility. For example, if an accounting procedure produces strange results, the users search for an error in the procedure; if a simulation program produces anomalous results, the user may have discovered an interesting finding; however, in this case, it is also necessary to determine whether the finding is actually valid, and not the product of a coding error;
(c) The difficulty of systematically exploring the entire set of possible hypotheses in order to infer the best explanation, in accordance with the previously discussed practice of abductive reasoning. This is mainly due to the inclusion of behavioral rules for the agents within the hypotheses, which produces a space of possibilities that is difficult if not impossible to explore completely.

The difficulty of communicating the results, which is described in (a), can be overcome by the diffusion of standardized tools to develop agent simulation models and by the introduction of a protocol to be applied to those tools. The first example, introduced in the mid-1990s, is Swarm (www.swarm.org), a project that started within the Santa Fe Institute, but then grew independently. Swarm is not a program in the classic sense, but a library of functions to build agent-based computer models. More specifically, it is a library of particular functions that are useful in the handling of a collection of agents, populating spaces with agents, or organizing events in time. Swarm is appropriately a milestone in simulation, thanks to the protocol suggested for using those functions, initially combining them with codes written in Objective C (a language combining C and a subset of SmallTalk) and, subsequently, in Java. The Swarm development team's original purpose, which was to create a *lingua franca* for agent-based model development, has only been partially achieved if one considers only the library of functions. With modern languages such as Python, a large part of the Swarm library is now unnecessary due to the facilities offered by the language itself. On the contrary, when considering the protocol aspect of the project, Swarm has been highly successful, being that protocol intrinsically the basis of several recent tools. For interesting considerations for the use of Python in agent-based programming, refer to Isaac [8] and for an application of the Swarm protocol to Python, see SLAPP, which is introduced here.

Many other tools have been built upon the Swarm legacy, such as Repast, Ascape, JAS, and now SLAPP. Different protocols are used by important tools, such as NetLogo and StarLogo. StarLogo TNG, a recent innovative version of StarLogo, is programmed by moving small shaped cards which fit together to build a running code. A second important innovation of StarLogo TNG was the production of animations that are very similar to animations in video games. This was done because video games are typically easily understood.

We can deal with the second weakness introduced in (b), i.e., the risk of using code with "bugs" that corrupt the results, both when employing the standard tools reported here (but this is in some way insufficient) and duplicating the code using two independent tools programmed by two different scholars. The result is never the same, due mainly to the use of random numbers when determining sequences. However, if the emergent phenomena are substantially similar in both constructions, we can be reasonably sure that the results are not the product of coding errors. This significant amount of work is suggested mainly for important and critical applications.

The third weakness described in (c), i.e., the difficulty of exploring the whole set of possible hypotheses (including the behavioral rules of the agents, where the full rationality and perfect information hypotheses are only one of the possible choices and not plausible) is determined by the uncontrolled dimension of the space of possibilities. This space of possibilities, when analyzed in a detailed way, is necessary for computations where no black box is allowed, although it generates an unmanageable set of possible paths. This is precisely why this paper proposes the use of neural networks to generate behavioral rules in an automatic way, or, in other words, the reinforcement of learning to extract the same rules from experience. In effect, this paper seeks to introduce strategies for going from the wide search of hypotheses about behavior to a procedure to calculate artificially generated, but plausible, rules.

Generating behavioral rules to achieve the capability of emulating cognition is a step that is both highly difficult and challenging. Consider Sun [14, p. 17]:

> What makes computational social simulation, especially computational cognitive social simulation (based on detailed models of cognitive agents), different from the long line of social theories and models (such as utility theory and game theory) includes the fact that

> it enables us to engage with observations and data more directly and test various factors more thoroughly. In analogy with the physical sciences (...), good social theories may be produced on the basis of matching theories with good observations and data. Cognitive agent based computational social simulation enables us to gather, organize, and interpret such observations and data with cognition in mind. Thus, it appears to be a solid means of developing social-cognitive theories.

Finally, with regard to the presentation of the results, the interaction between artificial agents and actual people, and the tools quoted above, it is useful to consider use of artificial on line worlds, such as Second Life (Bainbridge [5]).

The motivation for using ABM techniques with learning capabilities in this work is to explore and discover the consequences of self-generated behavioral schemes applied to an actual case of complex interaction (like the case of the diffusion of innovation and ideas in which many counterintuitive consequences of planned action occur in reality).

Let us conclude with Lave and March [10, p. 10]: "The best way to learn about model building is to do it." This applies to the metaphoric chameleons' model of innovation.

2 From a "Classical" Protocol to a New Tool

2.1 The Swarm Protocol

As seen above, the Swarm protocol is a "classical" reference in the relatively young world of the agent-based simulation, mainly for social sciences. The Swarm project, born at Santa Fe Institute, has been developed with an emphasis on three key points (Minar et al. [11]): (i) Swarm defines a structure for simulations, a framework within which models are built; (ii) the core commitment is to a discrete-event simulation of multiple agents using an object-oriented representation; (iii) to these basic choices Swarm adds the concept of the "swarm," a collection of agents with a schedule of activity.

The "swarm" proposal was the main innovation coming from the Swarm project, diffused as a library of function together with a protocol to use them. Building the (iii) item required a significant effort and time in code development by the Swarm team; now using Python we can attain the same result quite easily and quickly.

To approach the SWARM protocol via a clear and rigorous presentation it is possible refer to the original SimpleBug tutorial (Langton, 1996?), developed using the Objective-C programming tool (built on C and Smalltalk, www.smalltalk.org) by Chris Langton and the Swarm development team; the tutorial also has explanatory texts in the README files of the main folder and of the internal subfolders). The same contents have also been adapted by Staelin [13], to the Java version of Swarm, and by myself (Terna [16]), to create a Python implementation, exclusively related to the protocol and not to the libraries. Note that the Swarm original libraries are less important, anyway, using a modern object-oriented language. The SWARM protocol can be considered as a meta-*lingua franca* to be used in agent-based simulation models.

2.2 The Swarm-like Agent Protocol in Python (SLAPP)

The SLAPP project[2] has the goal of offering to scholars interested in agent-based models a set of programming examples that can be easily understood in all its details and adapted to other applications.

Why Python? Quoting from its main web page: "Python is a dynamic object-oriented programming language that can be used for many kinds of software development. It offers strong support for integration with other languages and tools, comes with extensive standard libraries, and can be learned in a few days."

Python can be valuably used to build models with a transparent code; Python does not hide parts, have "magic" features nor have obscure libraries. Finally, we want to use the openness of Python to: (i) connect it to the R statistical system (R is at http://cran.r-project.org/; Python is connected to R via the rpy library, at http://rpy.sourceforge.net/); (ii) go from OpenOffice (Calc, Writer, ...) to Python and vice versa (via the Python-UNO bridge, incorporated in OOo); (iii) do symbolic calculations in Python (via http://code.google.com/p/sympy/); and (iv) use Social Network Analysis from Python, with tools like the Igraph library (http://cneurocvs.rmki.kfki.hu/igraph/), the libsna library (http://www.libsna.org/), and the pySNA code (http://www.menslibera.com.tr/pysna/).

The main characteristics of the code reproducing the Swarm protocol in Python are introduced step by step via the on line tutorial referenced above. Summarizing:

- SimpleBug – We use a unique agent, running the time by the way of a *for* cycle, without object-oriented programming.
- SimpleClassBug – We run the time by the way of a *for* cycle, now with object-oriented programming to create and to use a unique agent as an instance of a class; in this way, it is quite simple to add other agents as instances of the same class.
- SimpleClassManyBugs – We run the time by the way of a *for* cycle, with object-oriented programming to create many agents as instances of a class; all the agents are included in a collection; we can interact with the collection as a whole.
- SimpleSwarmModelBugs – We run the time following a schedule of events based on a simulated clock; the schedule can be managed in a dynamic way (events can change sequences of events). With object-oriented programming we create families of agents as instances of classes, within a special class, the model class, that can be considered as a the experimental layer of our program.
- SimpleSwarmObserverBugs – As above, but we now have the model and all its items (agents, schedule, clock) included in a top layer of the application, that we name "observer". The observer runs the model and uses a schedule to apply tools like graphical representations, report generations, etc. The clock of the observer can be different from that of the model, which allow us to watch the simulation results with a granularity different from that of the simulated events.

[2] Python is at www.python.org. You can obtain the SLAPP tutorial files and the related examples at: http://eco83.econ.unito.it/terna/slapp.

In the object-oriented programming perspective the starting module generates the observer as an instance of the observer class. The observer creates: (i) the reporting tools, (ii) one or more models as instances of the class model and (iii) finally the schedule coordinating the top-level actions. Each model creates (iv) the instances of the agents and (v) the schedule controlling their behavior.

3 Creating an Imaginary Artificial World: the Surprising Behavior of Learning Chameleons

3.1 The Structure of the Chameleon World

A test model created with SLAPP involves intelligent chameleons. We have chameleons of three colors: red, green and blue. When two chameleons of different colors meet, they both change their color, assuming the third one. If all chameleons change to be the same color, we have a steady-state situation. This case is possible, although rare. Even so, what if the chameleons of a given color want to conserve their identity? On the other hand, what if they strongly want to change it?

With the on-line version of the chameleon model[3], we can see the chameleons moving randomly in their space. The chameleons can (i) move in a random way or (ii) refer to a *runner* mind (nine neural networks able to evaluate the moves from the point of view of the runners, used together) to avoid changing their color, or (iii) refer to a *chaser* mind (nine neural networks able to evaluate the moves from the point of view of the chasers) to look for contacts and so to change their color, if they want to change their color. As an example, if we tell a specific type of chameleons (i.e., the red ones) to be conservative, adopting the rules generated by a reinforcement learning process to avoid contacts, they become capable of increasing in number with the strategy of decreasing their movement when staying in zones free from chameleons of other colors, and getting close to subjects with their own color.

3.2 Reinforcement Learning

We use reinforcement learning Sutton and Barto [15] to develop intelligent behavior in our chameleons. Rewards and punishments come from the experience made in the past by chameleons while acting.

[3] The project, built in SLAPP, has been implemented also in NetLogo, relatively to the on-line visual representation of the results: with NetLogo you can easily produce an applet to be directly run in a browser; NetLogo is at http://ccl.northwestern.edu/netlogo/. The model is at http://eco83.econ.unito.it/terna/chameleons/chameleons.html, where you can find also an animation with voice explanations. I thank Riccardo Taormina, a former student of mine, for developing this kind of application with great involvement and creativity. Many thanks also to Marco Lamieri, a former Ph.D. student of mine, for introducing the powerful chameleon idea.

The evaluation of the rewards is quite simple. We consider here only a 3×3 space, as in Fig. 1, with nine potential moves, accounting also for staying in the initial patch. The red chameleon in the central position of the internal 3×3 square in Fig. 2 has three enemies around it; moving to the north in the center of the green square, it would have only two (visible) enemies, with a $+1$ reward; the reward can also be negative. Here we always consider two steps jointly and sum up their rewards, without the use of a discount rate. The double-step analysis compensates for the highly bounded rationality strategy applied both to the knowledge of the world (limited in each step to a 5×5 space) and the evaluation of the rewards, in a 3×3 area.

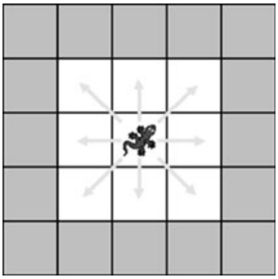


Fig. 1 The nine possible moves of the agent (staying in place and moving to one of the eight adjacent squares)

We have nine neural networks, one for each of the potential moves in the 3×3 space, with 25 input elements (the 5×5 space), ten hidden elements and a unique output, which is a guess of the reward that can be gained by choosing each one of the nine valid moves in the presence of each specific situation in the 5×5 space. Neural networks are used to summarize the results of the described trial and error process, avoiding the requirement of dealing with huge tables reporting the data upon the reward of each potential choice in each potential situation.

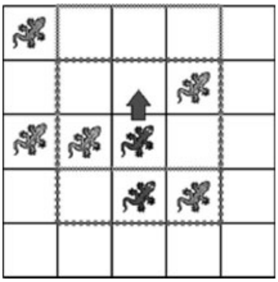


Fig. 2 Two subsequent spatial situations, always considered from the center of the two internal 3×3 squares

The dimension of the observed space, 5×5, obeys the bounded rationality principle. We could also consider a 7×7 space, but the number of potential situations increases enormously: in the case of the 5×5 space, omitting the central square, we have 224 potential situations. With a 7 × 7 space, we have 248 possibilities, a number on the order of 3 times 10^{14}. In the 5 × 5 case, we are able to explore a simplified space like that of Fig. 3, with about 17 million of possible states, with symmetric inputs not mandatory to produce symmetric outcomes (the decision system attributed to our agents is intrinsically imperfect).

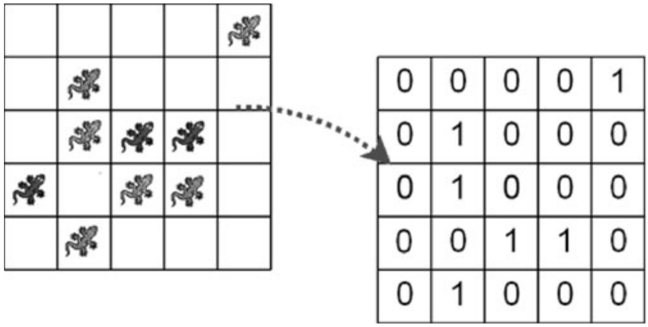


Fig. 3 The chameleon in the center of the grid (a red one) creates the matrix corresponding to its observed world. Values of 1 identify the places of the chameleons of colors other than red

3.3 Results in Chameleon Behavior

The chameleons can (i) move in a random way, or (ii) refer to a *runner* mind (the nine neural networks able to evaluate the moves from the point of view of the runners, used together) both to avoid contacts and to avoid changing their color, or (iii) refer to a chaser mind (the nine neural networks able to evaluate the moves from the point of view of the chasers, used together) to look for contacts and to change their color.

The running model can be found at the address reported in the note introducing the section, with and animation with voice instruction to train users in interacting with the simulation interface. It is possible to reproduce the different behavior of both the running and of the chasing chameleons, remembering that the actions of the chameleons are arising from an automatic learning process.

The model can also be metaphorically interpreted in the following way: an agent diffusing innovation (or political ideas or financial innovative behaviors) can change itself through interaction with other agents. As an example, think about an academic scholar working in a completely isolated context or, on the contrary, interacting with other scholars or with private entrepreneurs to apply the results of her work. On the opposite side, an agent diffusing epidemics modifies the others without changing itself.

4 Recreating an Actual World in an Artificial Way: Interbank Payments

4.1 The Payment Problem

From the imaginary world of the chameleons, always using SLAPP, we shift to focus on the concrete aspects of an actual banking system, recreating the interaction of two institutions (a payment system and a market for short-term liquidity) to investigate interest rate dynamics in the presence of delays in interbank movements.[4]

The problem is a crucial one because delays in payments can generate liquidity shortages that, in the presence of unexpected negative operational or financial shocks, can produce huge domino effects (Arciero et al. [2]). Here, we use agent-based simulation as a magnifying glass to understand reality.

4.2 Two Parallel Systems

We have two parallel and highly connected institutions: the RTGS (Real Time Gross Settlement payment system) and the eMID (electronic Market of Interbank Deposit). Considering the flow of interbank payments settled via the first institution, we simulate delays in payments and examine the emergent interest rate dynamics in the eMID. In this kind of market the interest rate is the price. A few microstructures of the market should be investigated and understood.

In Fig. 4 we present a modified representation of the standard sequence diagram of the UML (Unified Modeling Language, www.uml.org) formalism, introducing time as the first actor or user in the sequence. Events come from a time schedule to our simulated environment; the treasurers of the banks, acting via the RTGS system, with given probabilities bid prices, to buy liquidity in the eMID, or ask prices, to sell liquidity in the same market. Bid and ask probabilities can be different. The simple mechanism of bidding or asking on a probabilistic basis (if and only if a payment has to be done or has been received, as in Fig. 4), will be integrated – in future developments – with an evaluation of the balance of the movements in a given time period.

The different sequences of the events (with their parallel or immediate diffusion, as in Fig. 4) generate different lists of proposals into the double-action market we are studying. Proposals are reported in logs: the log of the bid proposals, according to decreasing prices (first listed: bid with the highest price); and the log of the

[4] I am deeply grateful to Claudia Biancotti and Leandro D'Aurizio, of the Economic and Financial Statistics Department of the Bank of Italy, and to Luca Arciero and Claudio Impenna, of the Payment System Oversight Office of the Bank of Italy, for having involved me in considering this important problem. The model is a contribution that I hope can be useful in identifying some aspects of the problem in a complexity perspective. You can obtain the code of the model by e-mailing the author.

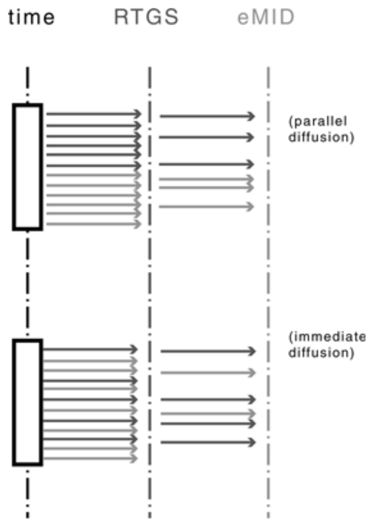


Fig. 4 Events to RTGS and from RTGS to eMid, in two different ways: a "quasi UML" representation of a sequence diagram

ask proposals, according to increasing prices (first listed: ask with the lowest price). "Parallel" means that we are considering an actual situation in which all the treasurers are making the same choice at practically the same time.

In Fig. 5 we discuss how a new price is proposed to the market when we look at the last executed price as a reference point, placing a price below it to get a ask position easily matched. In this case, both the case of parallel proposal and that of

Fig. 5 Looking at the last executed price, both in a parallel and in an immediate diffusion scheme

immediate diffusion are figuring out close expected situations. On the other hand: (i) this is not the case in the logs of the unmatched proposals, with ask prices greater than bid prices; (ii) the behavior of a market maker, not present here, is based on positive ask minus bid price differences. Other potential microstructures have to be investigated.

In Fig. 6, a new price is proposed to the market looking at the best proposal in the opposite log as a reference point, placing a price below it to get an ask position easily matched. The cases of parallel proposal and that of immediate diffusion are now producing quite different effects.

Fig. 6 Looking at the best proposal in the opposite market log, both in a parallel and in an immediate diffusion scheme

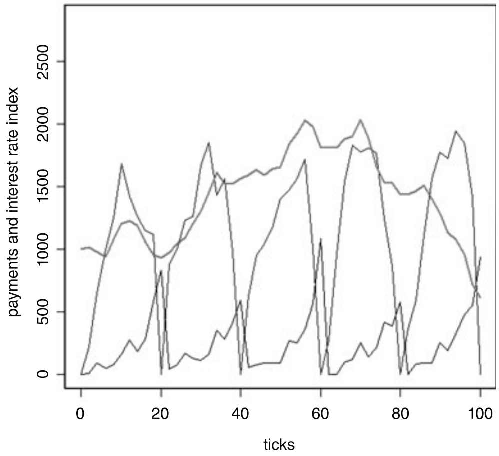
4.3 A Case of Simulated Behavior

We show here an interesting case of the dynamics emerging from this simulation environment, that occurs when the diffusion of the payment into the RTGS system is parallel and the operators look at the last executed price in the eMID. The run reported in Fig. 7 shows a non-trivial behavior of the interest rate. The dynamic is here magnified due to the dimension chosen for micro-movement in bids and asks. In these five days, we have a huge movement of this time series, as a consequence of significant delays in interbank payments. The simulation runs step by step, but we account for breaks in the time to reproduce the end of each day (i.e., cleaning all the positions, etc.).

Elaborating the interest rate series with the standard AR (autoregressive) and MA (moving average) technique, directly connecting SLAPP to R as seen above, we find in the graphs of the second row in Fig. 8 a typical $AR(1)$ model.

On the contrary, in a situation like that of Fig. 9, with data coming from a simulation run in which no payment delays occur, we find a random-walk dynamic in the interest rate first differences (first graph of the second row), without any correlation evidence.

Fig. 7 Interest price dynamic (*upper line*), stock of due payments (*intermediate line*) and flow of the received payments (*lower line*) in case of relevant delay in payments (with a uniform random distribution between 0 and the 90% of the available time until the time break). Time breaks at 20, 40, ... (end of a day)



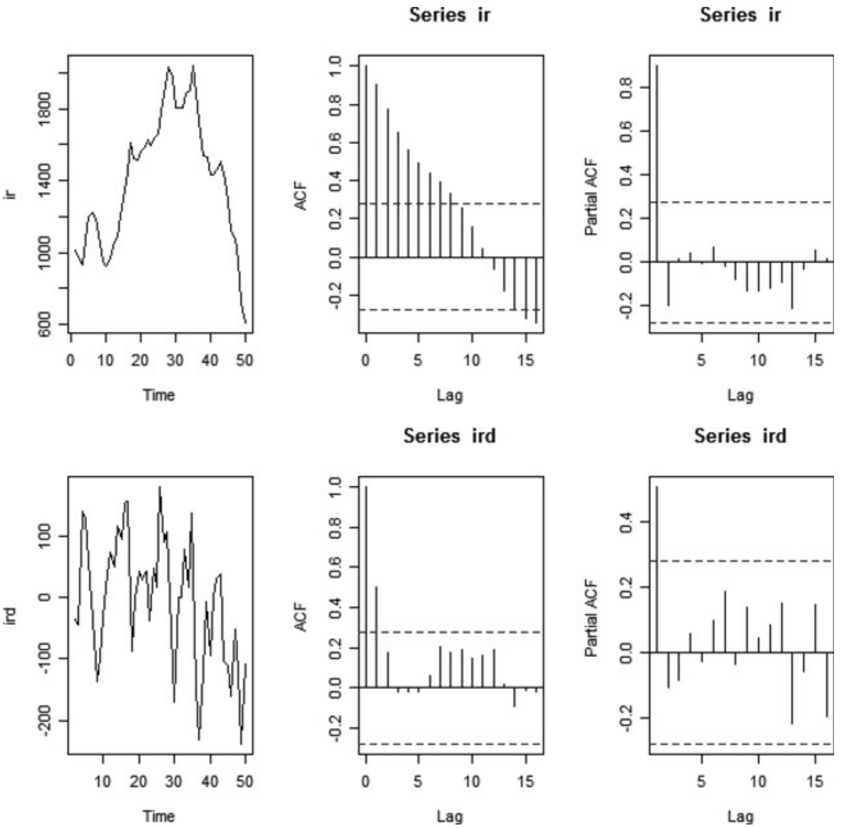


Fig. 8 The autocorrelation analysis of the interest rate data of Fig. 7 (with the presence of delays in interbank payments). *First row*: raw data; lagged correlations among data; the same, but as partial correlations. *Second row*: data first differences with lag 1; their lagged correlations; their lagged partial correlations

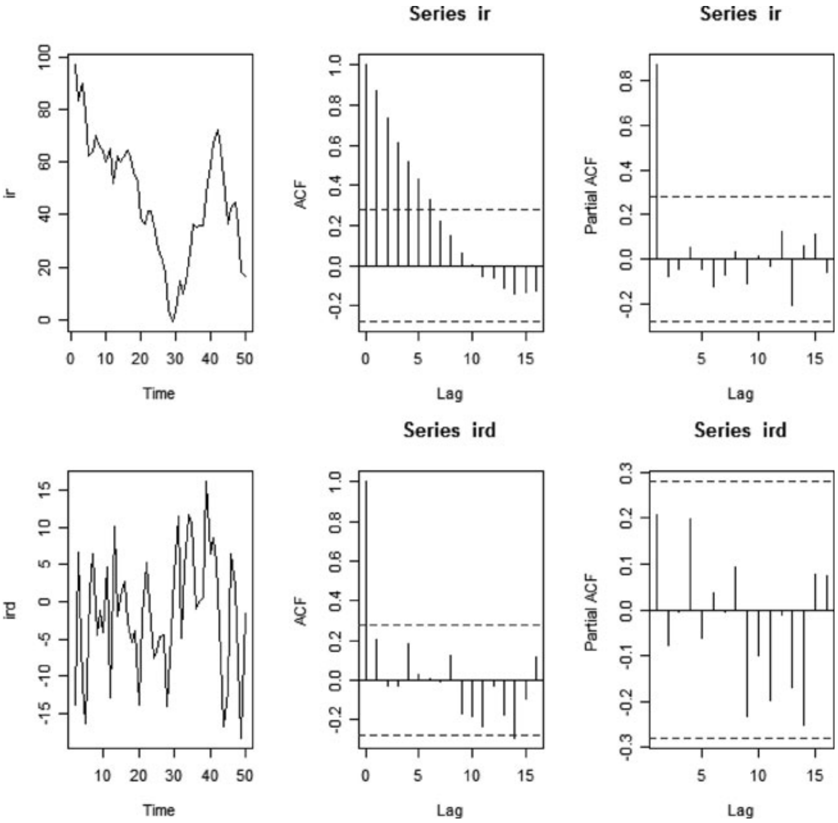


Fig. 9 The autocorrelation analysis of the interest rate data in a case of absence of delays in interbank payments). *First row*: raw data; lagged correlations among data; the same, but as partial correlations. *Second row*: data first differences with lag 1; their lagged correlations; their lagged partial correlations

This analysis suggests that the strong difference between these two situations is the direct consequence of the presence/absence of the payment delays.

Future Developments

There are three promising lines for future developments:

- In terms of SLAPP, the development of the capability of directly probing each agent, the graphical representation of spatial dynamics and of social networks links, and the simplification of the schedule code for event dynamic.
- In terms of chameleons, the introduction of communication capabilities, mainly via information left in the space, to search for the emergence of social coordination and of social capital.

- In terms of the payment system, applying also in this case the reinforcement learning technique, the introduction of a market maker, i.e., a subject continuously asking and bidding a price for selling and buying money, with a spread, assuring liquidity to the market and developing a pricing capability aimed at the containment of liquidity crisis.

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Exploration Modes and Its Impact on Industry Profitability

The Differentiated Effects of Internal and External Ways to Access Market Knowledge

Lucio Biggiero

Abstract. In the behavioural and evolutionary perspectives it is usually supposed that firms' performance depends significantly on their exploration activities, because they enlarge firms' knowledge base. The analysis has been here focused on accessing external knowledge, that is acquiring and managing others' experience, and on developing internal knowledge, that is accumulating and using the own direct experience. The effects of these two exploration modes on profitability at industry and segment level with honest and dishonest agents has been investigated through an agent-based model of industry, whose firms, in order to achieve competitive advantages, look for the best suppliers. Firms are characterized by bounded rationality in terms of absorptive capacity and organizational memory. By testing six groups of hypotheses it is shown that indirect experience has the main positive impact, especially in contexts of opportunist behaviours. It also improves performance in terms of the speed to reach the maximum profitability, but conditionally to the presence of high degree of access and management of direct experience, and at the price of high instability of the collective outcomes. Even more interestingly, it clearly emerges that, even if submitted to almost the same rules and size, the industry segments of final producers and first tiers perform quite differently. Moreover, the two exploration modes and opportunism impact very differently on each industry segment. Consequently, their effects on single firms' or collective profitability cannot be understood if it is not specified the specific segment where such firms are located. Moreover, the outcome at industry level strictly depends on industry structure and on the competitive pressure among suppliers and on the ratio of transactions occurring between each supplier–buyer pair.

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M. Faggini, C.P. Vinci (Eds.), *Decision Theory and Choices: a Complexity Approach*

1 Introduction

During eighties and nineties knowledge and learning have been posed at the core of reflections in organization science and evolutionary economics. The resource-based view and its development into the knowledge-based view collect the most relevant literature. Though not clearly and conclusively demonstrated by empirical research, it is argued that organizational performance strictly depends on organizational knowledge and learning. In particular, it is highlighted that out of the mainstream economics heaven, where agents have unbounded or extremely high rationality (and thus, unbounded knowledge), industry and firms competitiveness depends also on exploration capacities (Cyert and March, 1963; March, 1988, 1991; Simon, 1991). Instead of matching the ideal requirements of Bayesian probability updating and regression estimation, agents' learning likely occurs by means of rule or routines following, represented by their (more or less intentional) decision making processes (March, 1988, 1997).

Among the many forms of organizational learning, after March's seminal paper (1991) the attention has been focused largely on the characteristics and the difficult balance between exploration and exploitation. Exploration includes search, flexibility, innovation, experimentation, variety, etc., while exploitation concerns decisions, replication, repetition, routines, etc. Though these lists contain a broad set of activities, instead of analyzing its differential contributions to firm's performance the academic debate proceeded mostly by applying the original labels and contents to a vast set of fields.

Here it is proposed to modify this approach because of two reasons. Firstly, the broadness of the two categories of exploration and exploitation favoured a certain confusion and ambiguity (Gupta et al., 2006; Li et al., 2008), far beyond the natural transformations of a fertile approach as that triggered by March's paper. For instance, functional, firm or industry level of analysis can lead to quite different theoretical definitions and operationalizations for empirical research. Therefore, a first problem is that the two categories of exploration and exploitation are so broad that its various components could behave in a so different manner that it would be hard to discern its specific contributions. Secondly, in March's approach and most of the following contributions it is not clear why decision making should be placed only in the exploitation side, because both sets of activities require it. Do we explore in this or that direction? Do we test (engage, experiment) this or that supplier?

This paper innovates current literature in a number of ways. Firstly, the focus is put only on exploration, which is investigated by distinguishing and analyzing two of its main activities: the enlargement of knowledge base through indirect and direct experience. Hence exploration is distinguished in two modes: 1) searching, acquiring (and using) new knowledge residing by competitors; and 2) building new knowledge by engaging a larger number of suppliers. Secondly, decision making works in both the exploration modes and it entails the simplest learning mechanism of comparing current alternatives with the present memory of suppliers' quality and informers' reliability. This memory generates also agents' cognitive mapping (Daniels and Johnson, 2002; Eden, 1992; Fiol and Huff, 1992; Swan, 1997).

Thirdly, instead of considering only technological knowledge (be it patents, R&D, innovations, etc.), as it is usually done in almost all literature (Dosi et al., 2000; Foss and Robertson, 2000; Howells et al., 2003), the focus is on market knowledge (Li et al., 2008; Morgan et al., 2003), namely that referred to suppliers' characteristics in terms of product quality.

Fourthly, the knowledge about suppliers' quality is distinguished between that acquired through external and internal knowledge, that is through indirect and direct experience. Therefore, the use of the concept of external knowledge is quite different from that used in current literature (Bierly III et al., 2009). Likely, it would be better reformulated as a distinction between external and internal ways to access the knowledge about the others. In fact, one of the points here underlined is that future studies had to distinguish the knowledge *on* the interacting party from the knowledge that the interacting party brings or transfer. In other words, knowing something about a certain supplier is different than accessing what that supplier knows. The same holds respect to a potential partner for strategic alliances: knowing who possesses valuable knowledge is different information than knowing what knowledge she possessed. This difference is even more accentuated and relevant if the type of knowledge under scrutiny concerns relational behaviours, like being reliable or not. The ambiguity rests on the fact that, from the point of view of current literature, the acquisition of others' knowledge is considered external knowledge regardless of the ways it is accessed. Conversely, it should be noted that information *about* the possessors of knowledge (and limitedly also about their knowledge) is usually obtained through the transmission of others' own experience and reputation. In this sense, this type of information should be considered truly external knowledge. Conversely, the access of someone's knowledge requires usually and mostly a direct experience (relationship) with that one, and in this sense this can be considered a type of internal knowledge. In sum, even maintaining that others' knowledge is external knowledge, others' knowledge should be kept distinguished from the knowledge of the others. Both can be accessed by both external (indirect experience and reputation) and internal (direct experience) knowledge.

Moreover, and this is the last original aspect of the paper, the impact of knowledge is distinguished according to the cases in which information accessed and transferred were true or false, depending respectively on informers' honesty or dishonesty. This is due to the fact that false information can damage more than no information (Carley and Lin, 1997). However, again differently from current literature on exploitation-exploration, information is not exchanged because of some form of strategic alliance but just by asking competitors about what they know or learnt from their or others' past experience. This reflects a lighter mechanism of knowledge transfer respect to what is generally supposed to hold in strategic alliances (Gulati, 1998; Hagedoorn and Schakenraad, 1994; Mowery et al., 1996). This is plausible just because it is not involved a large amount of complex tacit or explicit knowledge, but rather a simple and synthetic judgment about suppliers' quality.

This type of knowledge circulation is recently addressed by the studies on industrial clusters and knowledge networks, which claim the role played by trade associations, buzz, pipelines, etc. (Gertler, 1995; Malmberg and Power, 2005; Maskell, 2001a, b; Storper and Venables, 2004; Tallman et al., 2004; Zook, 2004). It is underlined that besides formal or someway highly committing forms of knowledge transfer there are many "light" forms of information circulation. Being one of these, corporate reputation and the related literature extensively deals with the creation and transmission of more or less reliable judgments (Barnett et al., 2006; Bromley, 1993, 2002; Caruana, 1997; Fombrun, 1996, 2006; Fombrun and Shanley, 1990; Fombrun and Van Riel, 1997). Actually, indirect experience circulates in the forms of individuals' own experience or collective opinions collected by a single agent. The meaningful difference between the two is that in the former case the informer takes the responsibility of what she says, while in the latter case she does not.

By means of an agent-based simulation model (Carley and Prietula, 1994; Gilbert, 2008; Lomi and Larsen, 2001), the effects on industry profitability of the two exploration modes for choosing the best suppliers are analyzed in a three segments industry structure. Therefore, the level of analysis is not intra- but instead inter-organizational, namely the buyer–buyer and supplier–buyer ones (Hsuan Mikkola, 2003; Imrie and Morris, 1992; Lane, 2001; Webster, 2002). Four basic and some intermediate situations are compared by varying low and high dosages of internal and external knowledge. Moreover, the moderating role of agents' cheating attitude is considered too.

The paper proceeds as follows. In next section six groups of hypotheses are raised as concerning the effects of enlarging a firm's knowledge base on the level and stability of industry profitability in terms of average profit. These effects are further distinguished in a context of honest and opportunistic behaviors. In section three the simulation model is described and the parameters of the 32 virtual experiments are summarized. Then, in the next section results are discussed and hypotheses tested in terms of final outcomes after 400 simulation steps, and in terms of dynamic patterns.

2 The Hypotheses

It is commonly acknowledged that trust is a crucial competitive factor at both firm and industry level (Bachmann and Zaheer, 2006; Lane and Bachmann, 1996, 1998), and on the opposite that lying is one of the main forms of opportunism (Williamson, 1996). A huge theoretical literature suggests that opportunism or lack of trust, which indeed are far from being univocally defined, damage collective performance (Child, 2001; De Jong and Nooteboom, 2000; Gambetta, 1988; Hall, 1992; Humphrey, 1998; Kramer and Tyler, 1996; Lane, 1995; Lane and Bachmann, 1996, 1998; Sako, 1998). These theoretical claims are usually confirmed by a number of empirical researches, though its methodological and conceptual differences make them quite incomparable. Recently, some simulation models added new insights on this issue (Biggiero and Sevi, 2009; Giardini et al., 2008; Klos and Nooteboom, 2001; Lin and Li, 2003; Prietula, 2001; Tychonov et al., 2008), but again with a certain difficulty in comparing results. One of these models (Biggiero and Sevi, 2009) demonstrates that agents' cheating behaviour severely damages industry profitability, and that this outcome holds for different industry sizes, though the extent of the profit loss increases less than proportionally with the increase of industry size. However, in that model agents' capacity to access others' information and to make experience into their own decision space were kept fixed, leaving unexplored whether these two factors could affect industry performance. Using the CIOPS model (Biggiero and Sevi, 2009) this is just the aim of this paper, which takes industry size fixed and leaves the two exploration modes varying.

According to most management literature (March, 1997; Simon, 1997), and behavioural, cognitive (Dosi and Marengo, 2007; Rizzello, 2003) and evolutionary economics (Langlois and Everett, 1994; Witt, 1993), agents' abilities are bounded. They have no complete knowledge of the environment, whose exploration depends just on those abilities. The difficult balance between exploration and exploitation (Levinthal and March, 1993; March, 1991; Gupta et al., 2006) affect agents' and industry competitiveness. However, both the categories of exploration and exploitation are very broad. Exploration, on which this paper is focused, includes activities like search, variation, risk taking, experimentation, play, flexibility, discovery, innovation (March, 1991:71). If all components moved towards the same direction, this crowded set would be just a useful approximation, but if, instead, these activities determined very different outcomes, then it would be necessary to make distinctions. Actually, they identify very different tasks and routines, located into different positions and functions within organizations (Mintzberg, 1973). Plausibly, a firm can have good capabilities in one of these activities and not in the others. Moreover, its costs and effects could combine in unexpected and nonlinear ways.

In a competitive environment where suppliers' quality is important (Olhager and Selldin, 2004) and it is unevenly distributed, clients' ability to find good suppliers represents a key factor of success. Information becomes a strategic resource not only for the final producer but for the whole supply chain (Chen, 1999; Chen et al., 2000; Lee et al., 1997). The own experience is of course an important source of information, which is realized over time through subcontracting. However, when the number of potential subcontractors is high, a single buyer cannot experience directly most of them. Moreover, if the average number of subcontractors per buyer, that is the individual supplier–buyer ratio were high, this mode of exploration would be very risky. Actually, buyers have also the possibility to exploit others' experience in the form of two types of information: 1) that related to others' direct experience, for which they take the responsibility as reliable informers; 2) that stemming from others' information, which is transmitted as such and for whose truth it is not assumed any responsibility. This latter is defined as reputation (Conte and Paolucci, 2002).

Though the two types of information differ as respect to the assumption of responsibility by the informer, from the point of view of the questioner they are both forms of indirect experience. Therefore, in order to explore that part of competitive environment represented by suppliers, clients can activate two basic exploration modes: direct and indirect experience, which is further distinguished into responsible information and reputation[1]. This latter is passed as such, and thus, it is a pure form of indirect experience-based information (Mason-Jones and Towill, 1998, 1999). The two exploration modes enhance two corresponding types of learning processes (Argote, 1999), respectively based on internal and external knowledge.

Now, a key issue is that asking judgements to somebody is usually far less costly than engaging into subcontracting and eventually coping with bad suppliers. Moreover, firms could have very different social capital, and thus very different absorptive capacity (Cohen and Levinthal, 1990): for instance not all of them can have a purchasing office. This problem holds in supply chain management too (Arnulf et al., 2005; Tan, 2001). Therefore, if there is no sound reason to argue that the enlargement of knowledge base through direct and indirect experience has the same effects on performance, it is quite reasonable to suppose that its effects are different, and eventually combined in a nonlinear way. Hence, the first group of hypotheses sounds as follows:

Hypothesis 1a Internal and external knowledge contribute differently to improve performance.

Hypothesis 1b Internal and external knowledge combine its effects in a nonlinear way.

Due to information asymmetry (Akerlof, 1970; Mas-Colell et al., 1995; Stigler, 1961) and their economic (Williamson, 1975, 1981, 1985, 1994, 1996) and social nature (Carley and Newell, 1994), under some circumstances agents cheat or betray previous agreements. An extant literature highlights the negative effects of these forms of opportunism in general (Child, 2001; Gambetta, 1988; Hall, 1992; Humphrey, 1998; Kramer and Tyler, 1996) and for the supplier–buyer relationships (Bessant et al., 1994; Doney and Cannon, 1997; Lane, 1995, 2001; Lane and Bachmann, 1996, 1998; Webster, 2002; Zaheer et al., 1998). Hence, it is plausible to argue that cheating damages industry profitability regardless of the specific mode of exploration used by agents to find the best suppliers. Therefore, it can be suggested that:

Hypothesis 2a In industries characterized by cheating agents performance is significantly lower than in industries with honest agents, even if their knowledge base built on indirect experience enlarges considerably.

Hypothesis 2b In industries characterized by cheating agents performance is significantly lower than in industries with honest agents, even if their knowledge base built on direct experience enlarges considerably.

In most economics and management literature it is generally acknowledged that more knowledge leads to better performance. Indeed, the strict connection between

[1] This distinction is not so important in this paper, but it would be critical once the simulation model included also agents' retaliation strategies against unreliable informers, because it would be plausible to react only to responsible and not irresponsible information.

knowledge and performance has not been clearly and conclusively demonstrated (Carlucci and Schiuma, 2006), if not for partial types of knowledge and performance indicators, like those concerning patents and R&D accumulation and access (Antonelli, 1999; Bierly and Chakrabarti, 1996; Henderson and Cockburn, 1994; Pavitt, 1987; Powell et al., 1996; Stiglitz, 1987). However, bearing in mind the limits of learning (Levinthal and March, 1993), it is quite plausible to believe that learning depends on knowledge growth (Dosi et al., 2000; Grant, 1996; Nonaka, 1994).

Knowledge is here not meant just as a set of meaningful information, but it involves also its management and use for decision making. In fact, our agents: i) gather information from others by asking them who are the best suppliers according to their own experience or to the reputation they have accessed; ii) compare this information with their own experience; and iii) choose the highest values taking into account informers' reliability and suppliers' availability. Though all these abilities are practically indicated by the number of questions agents can handle, its role is much more important. These abilities are a crucial part of agents' rationality, because the knowledge they allow to access, its use through decision making processes, and its verification through suppliers' engagement contribute to build agents' cognitive maps and to define their learning capabilities.

In this specific model such capabilities are constrained by the number of questions agents can ask and by the forgetfulness effects (see next section). Moreover, the exploration of the competitive environment is limited to identify suppliers' quality and informers' (competitors') reliability, which is an important part of market knowledge. The extent to which agents access that knowledge represents their absorptive capacity (Cohen and Levinthal, 1990). In fact, though the concept of absorptive capacity has been investigated only with reference to technological issues, there is no reason to keep such a limitation (Zahra and George, 2002). For it concerns indeed all the types of knowledge firms can access and use, it can be measured here in terms of the number of questions agents can ask, manage and use for their decision making processes.

Since all the literature streams on organizational knowledge, learning and absorptive capacity underlie the fundamental positive role played by the access and use of external knowledge, it can be raised the following:

Hypothesis 3a A substantial increase of external knowledge improves industry profitability.

Cognitive agents (Carley, 1986, 1989; Carley and Newell, 1994; Sun, 2005) have not only the property of bounded rationality, but also that of manipulating information according to their convenience or social habit. Therefore, they can have different inclination to trust and lie (Child, 2001; Prietula, 2001), and these inclinations are especially crucial for supplier–buyer relations (Bessant et al., 1994; Lane, 2001). If cheating is intended as a gap of true information, then more knowledge should help more in presence of cheating rather than with honest agents, because it is supposed to reduce the lack of true information. This disclosure occurs indeed through the discovery of dishonest informers (agents). This leads to suggest the following:

Hypothesis 3b Performance improvement due to external knowledge is more relevant in industries characterized by cheating rather than honest agents.

However, managing knowledge could improve agents' learning. They can build better cognitive maps (Daniels and Johnson, 2002; Eden, 1992; Fiol and Huff, 1992; Swan, 1997) about their competitive environment, and such maps should lead to more effective actions, which, in turn, become "experiments" into the real world. It is therefore reasonable to suppose that, ceteris paribus, more experiments improve the likelihood of agents' cognitive maps, and this way also their collective performance. Hence agents' performance improves because direct experience implicitly tests the expectations created through the various forms of decision making. Direct experience in exploring the set of suppliers is here operationalized in terms of outsourcing degree (Espino-Rodriguez and Padròn-Robaina, 2006). If the same volume of purchase is split among many suppliers, clients, access more knowledge concerning suppliers' quality, and this in turn allows them improving the new selections. The degree of outsourcing indirectly acts as a test of clients' cognitive maps likelihood, and so it helps addressing their future choices. Further, and analogously to what supposed for the exploration through indirect experience-based information, if cheating is intended as a gap of true information, then more subcontracting should help more in presence of cheating rather than honest agents, because it is supposed to reduce the lack of true information. This leads to suggest the following:

Hypothesis 4a A substantial increase of internal knowledge improves industry profitability.

Hypothesis 4b Performance improvement due to internal knowledge is more relevant in industries characterized by cheating rather than honest agents.

Let's now wonder whether the effects on profitability distributes equally – or at least similarly – between industry segments. In the present model they are indeed constituted by the same type of agents, who are governed by the same selection and behavioral rules, decision-making patterns and cognitive capacity and boundaries. However, there are two differences. The first one refers to the place where the selection process starts up: from downstream (final producers) to selecting and proposing orders to intermediate firms. Consequently, it could happen that, when the selection of the best suppliers is more difficult, the performance of the two segments will differ, likely disadvantaging the intermediate agents.

The second difference is represented by the number of agents in the two segments, because first tiers are double of final producers, and therefore there are *potentially* twice suppliers respect to clients that could be selected and activated by these latter. The same mechanism holds between activated first tiers and the potential second tiers, that is the agents operating into the upstream segment of raw materials. Moreover, when direct experience exploration occurs by splitting purchases among 2, 4 and 8 suppliers though, then the number of activated suppliers grows from 40 to 80 to 160 to 320, the number of potential suppliers does not change, remaining fixed to 80. And likely, if the clients' learning process works well, it should lead over time to select the best suppliers, that is at least no more than half of the potential ones. What actually dramatically grows is the number of transactions, that is of actual experience through subcontracting occurring between the segment of final producers (FPs) and first tiers (FTs) (and as well between first and second tiers). It happens that progressively the best suppliers will work for more than one client moving up FPs profitability. Plausibly, ceteris paribus this outcome had to be more accentuated as the number of subcontractors per client grows from 2 to 4 and 8. That is, as the exploration through direct experience increases, segments' performance is expected to differentiate. Finally, it is reasonable that segments' performance differences are less marked with dishonest agents, because in this case the positive effects of clients' learning processes cannot impact effectively. From all these considerations, the following hypotheses can be advanced:

Hypothesis 5a As the knowledge base extension occurs through direct experience, that is by increasing the number of subcontractors per client, FPs profitability markedly differs from that of FTs.

Hypothesis 5b As the knowledge base extension occurs through direct experience, FPs profitability becomes significantly superior of that of FTs.

Hypothesis 5c With dishonest agents FPs profitability is still superior to that of FTs, but with a reduced gap respect to what happens with honest agents.

If the analysis of profitability is moved to the dynamics instead of the final average value, it is plausible to expect that in a context of honest agents the maximum level is reached earlier than in a context of dishonest agents, because it is not necessary to discover false informers and to face with false information. Secondly, ceteris paribus a growth of knowledge base, regardless whether obtained through direct or indirect experience, had to speed that time. Thirdly, the difference between the two types of knowledge growth should play in terms of the stability of trends, because more direct experience amplifies the effects of possible errors, while more indirect experience had to smooth such wide oscillations. In fact, it allows firms to improve their knowledge of suppliers and informers without risk, that is without concretely experiencing suppliers' quality and informers' reliability. Hence, it is plausible to suppose that:

Hypothesis 6a The maximum level of profitability is reached earlier with honest rather than with dishonest agents.

Hypothesis 6b The growth of knowledge base, regardless whether obtained through direct or indirect experience, speeds the time required to reach the maximum level of profitability.

Hypothesis 6c When the knowledge base enlargement is obtained through direct experience, average industry profitability becomes much more unstable.

3 The Model Structure

A general overview. This study exploits the architecture of the CIOPS model (Biggiero and Sevi, 2009), which is a network of agents representing firms that interact through structural and cognitive links. The structural network is a productive system constituted by agents connected each other by (vertical) economic relationships, and interacting through orders and products. Conversely, information flows horizontally among agents of the same filiere segment. The industry is composed by 200 firms: 40 FPs, 80 *potential* FTs, and 80 *potential* second tiers. Sales and profits of FPs depend on the quality of FTs, which in turn depends on that of their suppliers (the segment of raw materials). It is supposed that clients always can sell products on the final market, and select subcontractors but not vice versa.

The cognitive links are related to: the type and amount of information available to agents, their rationality (computational capacity, forgetfulness effects, and levels of aspiration), their types of decision making processes, and their attitude to cheat. Agents' decision making process is constituted by *four types of choices that are compared each other in each step*: random choice, direct and indirect experience-based, and reputation-based. Clients ask other agents, information concerning suppliers' quality, and compare it with that existing in their memory, but since decisions take into account both present and past knowledge, agents learn and eventually improve their performance. This way they move within the information space, while their learning is undermined by some forgetfulness effects. Moreover, their rationality is bounded also by the limited number of information they can ask.

In sum, in the CIOPS model agents interact through structural and cognitive dynamic networks. Each agent develops his own cognitive map of both networks, and such representations are updated at every step according with the new knowledge acquired through direct and indirect experience, and through reputation.

A general view of the model (Fig. 1) shows its logical structure: FPs profits depend on the quality of their purchases, which in turn depends on suppliers' quality. Thus, FPs goal is to choose the best suppliers, but since FPs don't know suppli-

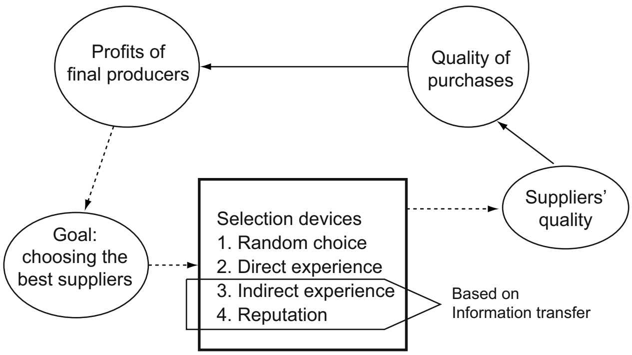

Fig. 1 The general structure

ers' quality, they have to select them through one of the four selection mechanisms: random choice, direct experience, indirect experience, or reputation. The latter two imply information transfer between competitors, who play also the role of informers.

Knowledge transfer mechanisms. Suppliers are selected according to their quality, which is verified only after transactions, because clients either could not know it in advance or could have correct or incorrect expectation, based on their memory information. Information reliability is not evaluated as such, but instead it depends on its source, that is informers' reliability. Actual suppliers' selection depends on clients' experience and cognitive characteristics: knowledge, computational capacity, expectations, and decision making processes.

Questioners' analysis of informers' trustworthiness leads to set up a list including only full reliable informers. If the number of informers entering the list is minor than the number of questions that the questioner is able to ask, the questioner asks also some unknown informer randomly chosen. The number of questions that the questioner is able to ask represents his absorptive capacity and enhances his knowledge access, which is examined at various levels: 4, 8, 16, and 39 questions asked and used. Into the segment of FPs they correspond respectively to accessing and managing 10%, 20%, 40%, and 100% of all the available information that competitors can transfer. Conversely, being FTs potentially double of FPs and being almost all of them activated through subcontracting, when FTs play the role of clients respect to their suppliers segment (that of second tiers), the corresponding levels of knowledge actually transferred represent only half of that potentially transferable.

Informers' trustworthiness depends on the truth of information that they previously passed. Unknown informers are assigned maximum trustworthiness value, as it is also for the required level of informers' reliability. Informers can tell the truth or falseness depending on their inclination toward cheating. True informers indicate the best supplier among the ones directly experienced, and the ones of which they know reputation. Conversely, false informers indicate the worst supplier as if it were their best. Regardless whether true or false, informers always specify the nature of their information, whether from direct experience or reputation. Moreover, there is no answer when informers have none of the two sources of information, that is, informers do not lie as concerning the possess or the natur of their information; that is, iformers do not lie as concerning the possess or the nature of their information.

As concerning reputation, it is acknowledged as such by a given questioner when his informers' evaluation of a given supplier converges on a strict range of values. Such an acknowledgement can be either contingent or implying questioners' or informers' memory. Therefore, reputation is formed by reaching a critical mass of converging judgments on a certain supplier, and so it is also transmitted as such from informers to questioners.

Direct experience comes from engaging subcontractors, and it also gives a feedback on informers' reliability and suppliers' quality. By collecting direct and indirect experience and reputation agents know suppliers and informers. The total knowledge residing into the industry and unevenly distributed among its agents is given by the four forms of memories about suppliers' quality through direct and indirect experience, and reputation, and about informers' reliability. This latter also help to choose reliable informers to be asked, and their answers feed the first three types of memory. It is noteworthy that, whereas decision space grows linearly with the number of suppliers, information space grows squarely with the number of informers. In fact, though not all potential informers are activated and consequently not all information are gathered in each step, potential informers are equal to $n(n-1)$ at each step, where n is the number of informers. Moreover, this information cumulates and in each step it is integrated by the information concerning direct experience, reputation autonomously created – that is not acquired as such from informers – and informers' memory.

However, all the types of memory are weakened by some forms of forgetfulness, which concerns information inactivated for long time: after 100 intervals of inactivated direct experience, 40 steps of inactivated indirect experience, and 60 of reputation the corresponding information dissolves. Similarly, informers non consulted for more than 60 steps disappear from questioners' memory.

Agents' characteristics and learning processes. Agents are boundedly rational, in that they have computational limits: they recognize only a limited number of agents, can ask only a limited number of questions, and suffer of forgetfulness effects. They are satisfiers and not maximizers, and exchange information only within and not between segments[2]. In order to choose their subcontractors, clients: i) scan their own memory on past direct and indirect experience with suppliers, ii) ask other agents as informers, iii) check the existence of any reputation built on the information passed by informers.

Agents are all equal in cognitive terms, and respect to goal seeking and expectations, and are provided with the same opportunities. They are equal also under structural respects, with the exception that first and second (source) tiers are randomly assigned a given quality, and that to FPs and FTs certain initial opportunities are randomly offered.

Agents differ over time on which type, how much and how reliable is their knowledge. Knowledge at industry level is just the sum of agents' knowledge, but not all knowledge circulates. For instance, in those configurations in which direct experience is prevalent, transferred knowledge is only a small portion. Noticeably, agents store and transfer different types and amount of knowledge, develop different cognitive maps of the others, and enact different cognitive networks.

As concerning trust and opportunistic behaviors, agents' attitude could be defined as "prudent trust" in the sense that they trust others but check their information and keep track of the corresponding result. Though they do not react through pure forms of retaliation, once an informer has been recognized as unreliable, they cancel him from the list of future potential informers. Hence, even if their reactions to cheating are "soft" and only passive, agents are not "blind trustees", because they check information and learn consequently. Moreover, it should be noticed that agents ac-

[2] For a detailed discussion of the decision making processes adopted in this model, see Biggiero and Sevi (2009).

quire free information, because it has no price and informers cannot refuse to give it. Finally, it should be taken into account that in this model all these cognitive operations are costless. In short, there are no direct costs of misplacing trust or checking information coming from indirect experience or reputation.

However, though there are no such direct costs, "prudent trust" has an indirect cost, that turns to produce effects on performance: at least in consequence of the first lie received from a cheating informer, the questioner is addressed to a bad supplier, and thus, his own performance is damaged. Being industry performance simply the sum of that of individual agents, industry profitability is negatively affected too. In this model reputation-based trust is more "insidious" and dangerous, because, for it does not imply informers' responsibility, the recognition of its eventual falsity does not lead to reject the informer who passed it. Therefore, a certain informer can again produce damages to the questioner.

By building and managing internal and external knowledge agents learn how to improve suppliers' selection and know unreliable informers. They learn by constantly confronting their current information with what they already know, but their rationality is bounded not only by the limited number of information they can ask, but also by the forgetfulness effects. These mechanisms make this model fully path-dependent and truly learning-based (Argote, 1999). However, agents' learning is always contingent, because it is not fixed into a specific state, but instead left to a continuous recalculation confronting current and stored knowledge. Agents do not "draw general findings" from their existing knowledge, but just use it for seeking their goals moment-by-moment.

Configurations of virtual experiments. Parameters and initial conditions used in these virtual experiments[3] are shown in Table 1. In each step a whole cycle choice/order/production/payment takes place. Supposing that it can represent a reality in which it lasts 5 working days, 400 steps describe 10 years of industry evolution from the very beginning. This could approximate a simple-product industry, whose production cycle is very short and can be realized completely in one week, but whose quality represents a competitive advantage. Medium-high quality segments of some consumer goods markets like clothing, footwear, leather, etc. could satisfy these characteristics. The time span consideration is very important to give sense to the results of dynamic patterns, because while in some cases performance stabilizes already before 50 steps, in many other situations it still remains uncertain after 400 steps. Sometimes performance becomes definitely unpredictable in the short run, while in the long run rounds on a mean value.

Indeed, complex products would better match the crucial role assigned suppliers' quality, as it is characteristic of biotechnology, aerospace, biomedicals, and most high-tech industries. However, they have production cycles extending far beyond two weeks and sometimes even beyond six months, and they require even long time

[3] The program running the model will be available on the URL of Knownetlab Research Center (www.knownetlab.it). To run the program is needed the platform and language LSD (Laboratory on Simulation Development), placed at www.business.auc.dk/lsd. The author is available to give any support to use it.

Table 1 Virtual experiments parameters

Structural parameters with constant values	
Filiere segments	3
Industry size	200 firms
Ratio FP/FT = 1/2	40 downstream firms, 80 suppliers in intermediate segment and 80 suppliers in upstream segment
Quality	Randomly uniform distribution between 0.5 and 1
Cognitive parameters with constant and uniform values	
Quality threshold	0.75
Clients' requirement of informers' reliability	1
Number of convergent information to form reputation	4
Number of intervals to forget inactivated direct experience	100
Number of intervals to forget inactivated indirect experience	40
Number of intervals to forget inactivated reputation	60
Number of intervals to forget inactivated informer	60
Varied structural parameters	
Outsourcing degree	1, 2, 4, 8, subcontractors for each client
Varied but uniform (among agents) cognitive parameters	
Number of questions	4, 8, 16, 39
Inclination to cheat	0 or 1

to define product and contractual characteristics. If such a real correspondence were given each step, it is clear that even 50 intervals would represent a very significant time span.

Since in sequential technology FPs are client of intermediate firms that on their own buy products from upstream suppliers, it is possible and interesting to keep distinguished FPs and FTs average profit. It is important to point out that: i) when required to deliver their products, FTs previously check whether they can find a satisfying and available supplier (second tiers) on their own. If not, they do not accept any engagement and stay inactive in that interval; ii) the rules to grow the two types of knowledge base hold for FTs too, because they act as clients respect to the segment of second tiers.

Experiments are executed varying: i) indirect experience, that is external knowledge or absorptive capacity, which is measured by the number of questions asked to informers (4, 8, 16, 39); ii) direct experience, that is internal knowledge, measured by agents' outsourcing degree (1, 2, 4 or 8 subcontractors for each client); and iii) agents' inclination to cheat (0 or 1), while keeping constant other parameters. Due to the high number of variables and the value that each variable may assume for each agent, within each virtual experiment suppliers differ only in quality while keeping constant other parameters, and clients (competitors) have the same quality threshold, and the same inclination to cheat. Other parameters not specifically discussed before are the following:

- quality threshold: this refers to agents' aspiration levels (Simon, 1982, 1997), who consider a supplier satisfying only if his quality is not lower than 0.75;
- clients' requirement of informers' reliability, which here it is supposed to be maximum, that is an informer is supposed to be credible only if he is completely reliable;
- reputation threshold, which indicates the number of convergent information to form reputation, which is fixed at 4 agents (including the questioner).

4 The Main Results

The first two hypotheses suggest that at industry level: (H1a) external and internal knowledge growths contribute differently to improve performance; and (H1b) they combine its effects in a nonlinear way. In order to test the first hypothesis we should look at the first column of the left side, and first and six row of Tables 2 and 3 respectively for the effects of varying only the external or the internal knowledge. As can be seen, any increasing of external knowledge does improve performance in the context of honest agents, and slightly reduces it in presence of opportunism. If we move at single segment level, we see (Tables 6 and 7) that this hypothesis is confirmed too, but, quite interestingly, the type of knowledge improving performance is the internal one, while the growth of external knowledge produces losses. Being the industry level just the pure mean of the two segments, the role played by the two types of knowledge in each segment results sharply opposite.

On the other hand the growth of internal knowledge lowers performance significantly and dramatically according to a context with honest and dishonest agents. Here the number of subcontractors per clients produces an opposite effect respect to what happens increasing external knowledge. A positive outcome occurs indeed moving from 1 to 2 suppliers per dishonest buyer. Performance improves sharply because agents know soon all false informers and bad suppliers, but at the same time clients' mistake are not so heavy as when there are 4 or 8 subcontractors per buyer. Therefore the first hypothesis is confirmed.

The other cells of Table 2 help testing the second hypothesis, because if we look at the performance variation due to the many combinations of external and internal knowledge growth it is evident that they occur in nonlinear ways. Neither the sign (positive or negative) nor the amount (respect to that of the two variables) of the variation show any proportional or even ordered rule. Hence, the second hypothesis is confirmed too, and this holds also at segment level.

The third and fourth hypotheses suggest respectively that in industries characterized by cheating agents performance is significantly lower than in industries with honest agents, even if their knowledge base enlarges considerably and regardless if

Table 2 Effects of knowledge enlargement on industry profitability

	Percentage increments respect to the default value								
	External knowledge growth						Internal knowledge growth		
	Honest agents								
	n of subcontractors						*n* of subcontractors		
Questions	1	2	4	8		Questions	2 to 1	4 to 1	8 to 1
8 to 4	0%	1%	3%	0%		4	–1%	–4%	–12%
16 to 4	0%	2%	6%	7%		8	0%	–2%	–12%
39 to 4	1%	2%	5%	7%		16	1%	1%	–6%
						39	0%	0%	–6%
	Dishonest agents								
	1	2	4	8			2 to 1	4 to 1	8 to 1
8 to 4	–1%	–1%	2%	22%		4	4%	–11%	–29%
16 to 4	–2%	0%	3%	20%		8	4%	–8%	–12%
39 to 4	–4%	7%	4%	21%		16	6%	–7%	–28%
						39	16%	–17%	–7%

Table 3 Effects of knowledge enlargement on industry profitability

	Percentage increments respect to the previous value								
	External knowledge growth						Internal knowledge growth		
	Honest agents								
	n of subcontractors						*n* of subcontractors		
Questions	1	2	4	8		Questions	2 to 1	4 to 2	8 to 4
8 to 4	0%	1%	3%	0%		4	–1%	–4%	–8%
16 to 8	0%	2%	3%	6%		8	0%	–2%	–10%
39 to 16	0%	0%	0%	0%		16	1%	–1%	–6%
						39	0%	0%	–6%
	Dishonest agents								
	1	2	4	8			2 to 1	4 to 2	8 to 4
8 to 4	–1%	–1%	2%	22%		4	4%	–15%	–20%
16 to 8	–1%	1%	1%	20%		8	4%	–12%	–4%
39 to 16	–2%	7%	1%	21%		16	6%	–12%	–23%
						39	16%	–17%	–7%

it is built on indirect (H2a) or direct (H2b) experience. Data show (Tables 4 and 5) that at industry level both the hypotheses are confirmed, because any configuration produces average profit losses, which ranges from 81 to 143, corresponding to 27% and 47% of the respective configuration. Hence, even if agents accessed all the knowledge that they are available to exchange reciprocally, in a context of oppor-

Table 4 Average profit at industry level

	Honest agents				Dishonest agents			
	Outsourcing degree (n of suppliers per single client)							
Questions	**1**	**2**	**4**	**8**	**1**	**2**	**4**	**8**
4	300	298	287	265	199	207	177	141
8	301	300	295	266	197	204	180	173
16	301	305	303	284	195	207	182	141
39	302	303	303	284	192	222	184	171

Table 5 Profit losses due to cheating behaviours

Questions	1	2	4	8
4	101	91	111	124
8	105	96	115	94
16	105	98	121	143
39	110	81	119	113

tunism such knowledge would be not enough to compensate the damage of cheating. Further, sometimes a larger amount of this false information worsens the outcome. On the other hand, excepted for the increasing between 1 and 2 subcontractors per client, further increases systematically worsen the performance.

At segment level (Table 10), FPs are much more damaged by cheating behaviours than FTs: the average loss among all configurations is 143 against 98 average profit. The other remarkable difference between the two segments is that FTs' losses are almost insensitive to the outsourcing degree, while FPs' losses increase almost 80% from 1 up to 8 subcontractors per client. Hence, cheating behaviours impacts very differently between the two segments and in relation to the degree of outsourcing.

Previous data allow testing also the third and fourth groups of hypotheses, which propose that both external (H3a) and internal (H4a) knowledge growth improves performance, and that the increments are more accentuated in opportunistic contexts (H3b and H4b). Before proceeding with the detailed comment, it should be noticed that the default value, that is the value corresponding to the configuration with only 1 subcontractor and 4 questions per client, is very near (95%) to the maximum. Here the surprising result is that even a substantial increase of external knowledge until all potential 39 informers are reached leaves industry profitability substantially unchanged. Moreover, it seems that significant increases of the internal knowledge almost always damages performance. Finally, it appears that both these effects occur regardless of agents' cheating attitude. These results would be inexplicable and definitely contrasting all theoretical and empirical literature if we didn't disaggregate the analysis at segment level, because the supposed advantages of acquiring more knowledge should be actually referred to single segments. In fact, buyers–clients (that is members–competitors of each segment, who at the same time are also questioners–informers) are the ones who directly benefit from enlarging their knowledge base. Now, the crucial issue is that what happens at industry level is so surprisingly because the performance of the two segments differs substantially and indeed diverge.

As shown by Table 5, in the default situation with honest agents the two segments perform in the same way, and they remain aligned and substantially unchanged even by merely increasing external knowledge. However, if the outsourcing degree grows, and even more if also absorptive capacity increases too, FPs perform much better while FTs much worst. While this difference is discussed below in more detail to test the fifth group of hypotheses, here it is necessary to move the level of analysis from the whole industry to the FPs segment (Tables 6 and 7), because we see that no any substantial increase of external knowledge improves profitability. On the contrary, it reduces performance in most configurations and especially (and markedly) with high degrees of outsourcing and when agents cheat. This is due to the double effects of higher impacts of eventual wrong choices and false information. When these effects are combined, the average profit loss can reach 24% respect to the default value. Hence, both the hypotheses H3a and H3b should be rejected.

On the contrary, the growth of internal knowledge through outsourcing improves profitability between 19 and 41% (Table 7), with the exception of two negative values occurring in the extreme cases of 8 subcontractors per client and the highest numbers of cheating informers. In these cases the effects of many casual wrong choices and a large amount of false information destroy performance. In fact, in the context of cheating agents profitability is lower than with honest agents, and anyway even in this context the contribution given by internal knowledge growth to profitability growth is decreasing (Table 6). Hence, while the hypothesis H4a is confirmed, the H4b should be rejected.

The discussion of the fifth group of hypotheses further develops the analysis at segment level. It was supposed that (H5a) as the knowledge base extension occurs by increasing the number of subcontractors per client, FPs profitability markedly differs from that of FTs. Data (Tables 8 and 9) confirm this hypothesis, because, for any supplier–buyer ratio higher than one-to-one, FPs profitability grows always upper 14% than that of FTs (with only one exception) regardless of agents' cheating attitude. Moreover, the gap enlarges when moving to higher ratios, and this confirms the next hypothesis too (H5b). Conversely, within a same ratio, that is by keeping constant the outsourcing degree, the growth of external knowledge does not improve significantly performance, and on the opposite very often depresses it, especially in contexts with dishonest agents. Finally, data confirm also the further hypothesis (H5c) that with dishonest agents FPs profitability is still superior to that of FTs, but with a reduced gap respect to what happens with honest agents.

Thus, FPs gain more by exploring through internal rather than external knowledge. Consequently, the modes of exploration matter for segment profitability, because the extension of profitability to many agents due to higher FP/FT ratios reverses the impact of the two modes. It makes at industry level exploration by direct experience less important than that by indirect experience, because FTs, who do not

Table 6 Effects of knowledge enlargement on FPs profitability

	Percentage increments respect to the previous value								
	External knowledge growth						Internal knowledge growth		
	Honest agents								
	n of subcontractors						*n* of subcontractors		
Questions	1	2	4	8		Questions	2 to 1	4 to 1	8 to 1
8 to 4	0%	0%	–1%	–3%		4	19%	6%	2%
16 to 4	–1%	0%	5%	6%		8	19%	4%	0%
39 to 4	1%	0%	–5%	–9%		16	20%	9%	2%
						39	20%	3%	–3%
	Dishonest agents								
	1	2	4	8			2 to 1	4 to 1	8 to 1
8 to 4	–2%	–2%	0%	4%		4	25%	–9%	–4%
16 to 4	–1%	–3%	–9%	–17%		8	25%	–7%	0%
39 to 4	–2%	13%	–3%	–11%		16	22%	–13%	–9%
						39	41%	–25%	–17%

Table 7 Effects of knowledge enlargement on FPs profitability

	Percentage increments respect to the default value								
	External knowledge growth						Internal knowledge growth		
	Honest agents								
	n of subcontractors						*n* of subcontractors		
Questions	1	2	4	8		Questions	2 to 1	4 to 2	8 to 4
8 to 4	0%	0%	–1%	–3%		4	19%	26%	28%
16 to 8	–1%	1%	3%	3%		8	19%	24%	24%
39 to 16	0%	1%	–2%	–6%		16	20%	31%	33%
						39	20%	23%	20%
	Dishonest agents								
	1	2	4	8			2 to 1	4 to 2	8 to 4
8 to 4	–2%	–2%	0%	4%		4	25%	13%	9%
16 to 8	–3%	–5%	–9%	–14%		8	25%	16%	16%
39 to 16	–5%	8%	–12%	–24%		16	22%	6%	–4%
						39	41%	5%	–12%

gain from the former mode, are many more than FPs. Bearing in mind that FTs play both roles, and that actually first and second suppliers do not exceed 80 agents in any configuration, the right way to describe this phenomenon is that, as the profitability of higher quantity of transactions per single client grows up, suppliers' performance

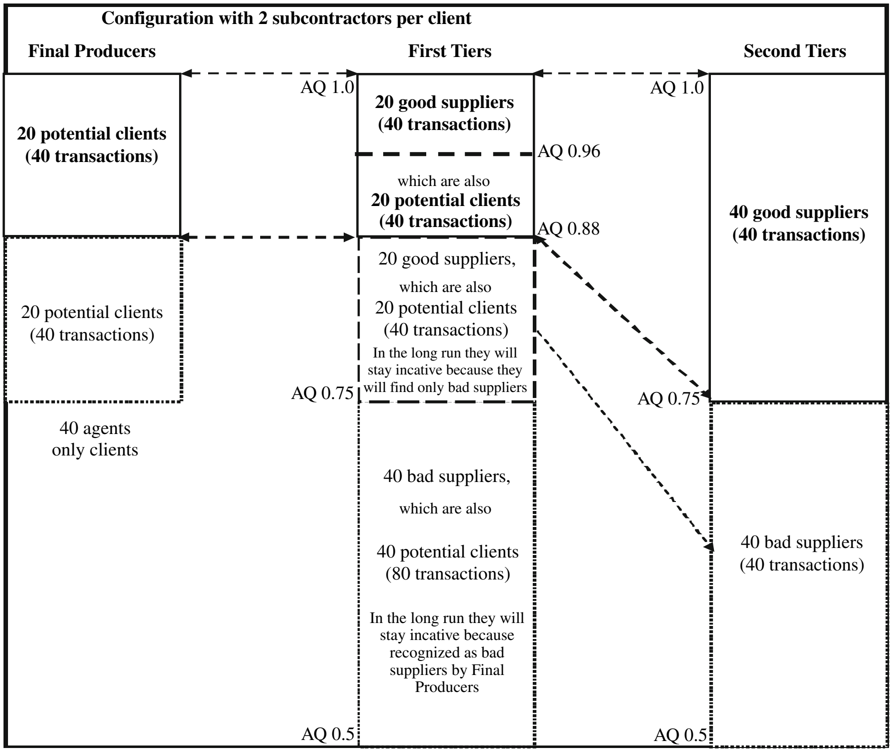

Fig. 2 Interaction processes between segments

becomes more important than clients' performance. Consequently, the mode of exploration more convenient for them decides for the whole industry.

What is further noteworthy is that high outsourcing degrees bring the average profit far beyond the maximum level corresponding to the default case, which can be assumed as that of just one subcontractor per client. It means that proportionally more exchanges do occur with good quality suppliers, but this is obtained at the price of reducing the quantity of active agents (Fig. 2). In fact, though FPs are half FTs, and therefore activate only half FTs, once FPs have more than one subcontractor they try to order products to all FTs. Before accepting, these latter check whether they can find satisfying and available suppliers. However, since second tiers are given the same quality distribution between 0.5 and 1.0, half of them are not good. Consequently, half FTs cannot work, and over time this tend to be the bad quality suppliers, because in the long run, if the selective process works well, only the good ones will be required to supply products. In conclusion, many FPs and FTs over time will not realize all their transactions, but the successful FPs will have a higher quality. In the case of two subcontractors per client, the final result of the complex feedback processes between agents with the pivotal role of FTs, who are at the same time single transaction suppliers and double transactions clients, is the following: 20

Table 8 Profitabilty at segment level (in absolute values)

	Final Producers				First Tiers			
	n of subcontractors							
	1	**2**	**4**	**8**	**1**	**2**	**4**	**8**
Questions	Honest agents							
4	301	358	379	386	299	268	257	248
8	302	359	375	376	301	271	269	251
16	299	360	391	397	302	277	274	267
39	302	361	373	361	302	274	279	273
	Dishonest agents							
4	197	246	224	216	201	188	161	131
8	193	242	224	224	200	186	166	166
16	192	233	203	185	199	193	175	135
39	188	265	198	165	196	201	179	172

Table 9 Comparisons of segments profitability

	FP-FT				FP/FT			
	n of subcontractors							
	1	**2**	**4**	**8**	**1**	**2**	**4**	**8**
Questions	Honest agents							
4	3	89	123	138	1%	25%	32%	36%
8	1	89	106	125	0%	25%	28%	33%
16	–3	83	118	130	–1%	23%	30%	33%
39	0	87	94	88	0%	24%	25%	24%
	Dishonest agents							
4	–4	58	63	85	–2%	24%	28%	39%
8	–7	56	58	58	–4%	23%	26%	26%
16	–7	40	28	50	–3%	17%	14%	27%
39	–8	63	19	–7	–4%	24%	10%	–4%

Table 10 Profit losses due to cheating behaviours

	Final Producers				First Tiers			
	n of subcontractors							
Questions	**1**	**2**	**4**	**8**	**1**	**2**	**4**	**8**
4	104	112	155	171	97	80	96	118
8	109	118	151	152	101	85	103	85
16	107	127	188	213	104	84	98	133
39	114	97	175	197	106	73	100	102

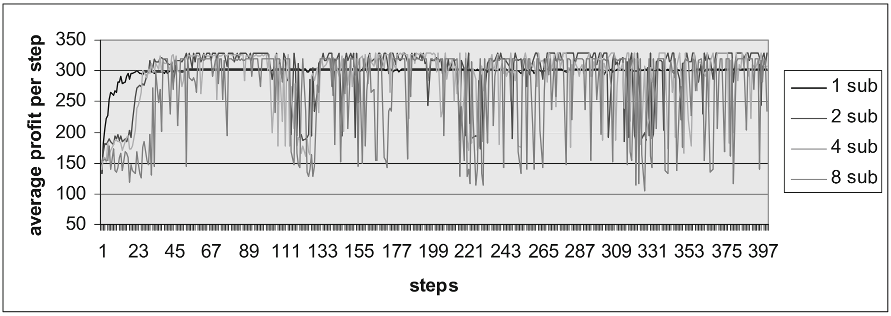

Fig. 3 Dynamics of average profit of honest agents at industry level with low indirect experience-based knowledge

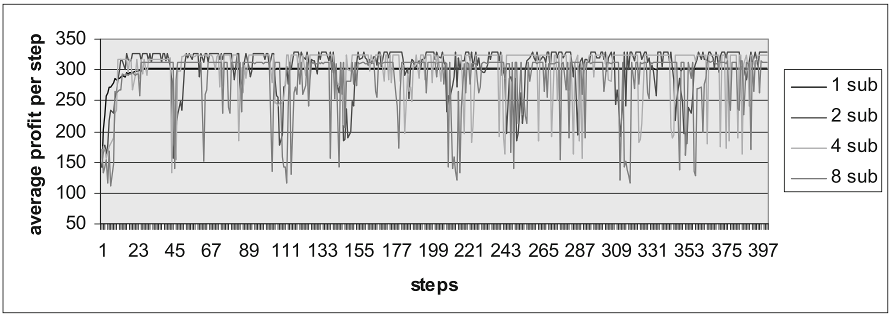

Fig. 4 Dynamics of average profit of honest agents at industry level with high indirect experience-based knowledge

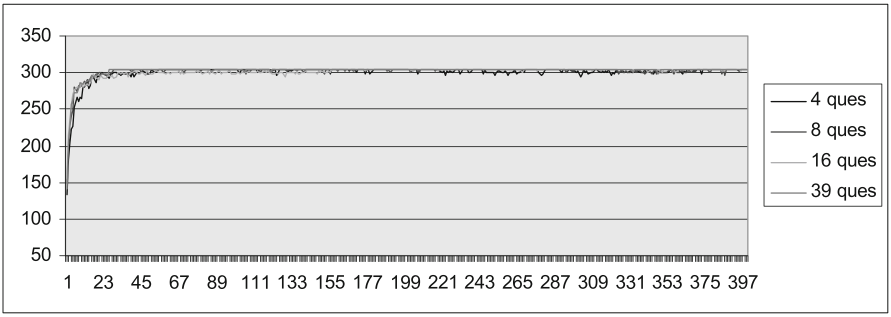

Fig. 5 Dynamics of average profit at industry level of honest agents with low direct experience-based knowledge

FPs and 60 FTs fail; FTs average quality keeps constant and that of FPs substantially increases.

The sixth group of hypotheses concerns the dynamics of industry profitability, which can be very different, even when the average quality at the 400th interval is very similar. Eight dynamic patterns have been distinguished (Figs. 3–10), four for

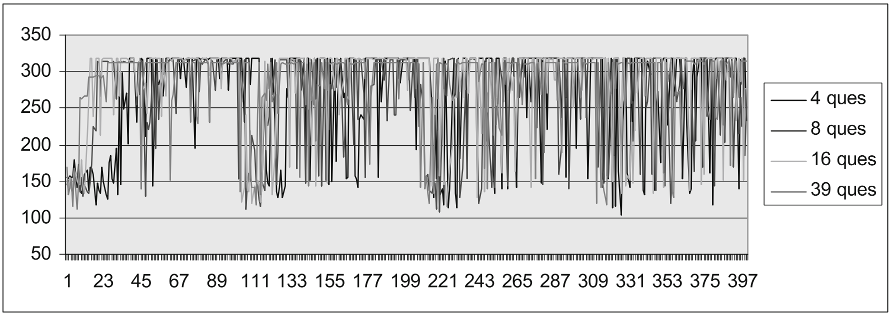

Fig. 6 Dynamics of average profit at industry level of honest agents with direct experience-based knowledge

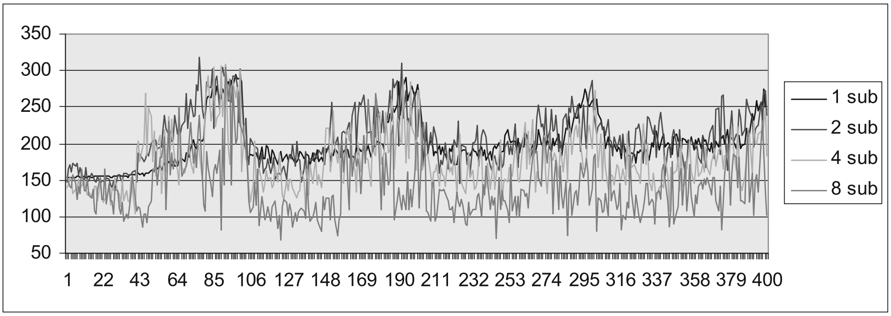

Fig. 7 Dynamics of average profit at industry level of dishonest agents with low indirect experience-based knowledge

the context with honest and four for that of dishonest agents: low and high external knowledge with variable numbers of subcontractors, and the symmetrical combinations. The first supposition (H6a) states that the maximum level of profitability is reached earlier with honest rather than with dishonest agents. Figures confirm this hypothesis, because in each combination between the two modes of exploration the context with honest agents records an earlier time to reach its maximum level of profitability.

The second hypothesis concerning dynamics patterns (H6b) suggests that a knowledge growth, regardless whether obtained through direct or indirect experience, had to speed the time required to reach the maximum level of profitability. This hypothesis is only partially confirmed, namely when at least one of the two modes of exploration is high. In other words it does not work if the knowledge growth occurs when agents have the lowest level of both internal and external knowledge.

The final hypothesis (H6c) proposes that when the knowledge growth is obtained through direct experience the average industry profitability becomes much more unstable. Figures fully confirm this hypothesis, which actually explains also the very different performance between FPs and FTs, and the wide difference between honest and dishonest agents. In fact, the effects of chaotic dynamics is

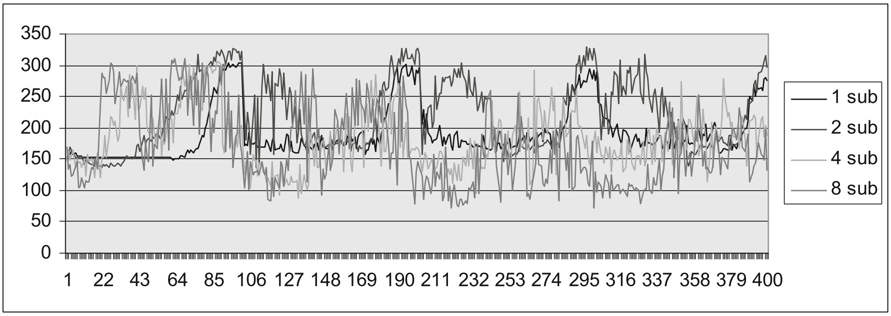

Fig. 8 Dynamics of average profit at industry level of dishonest agents with high indirect experience-based knowledge

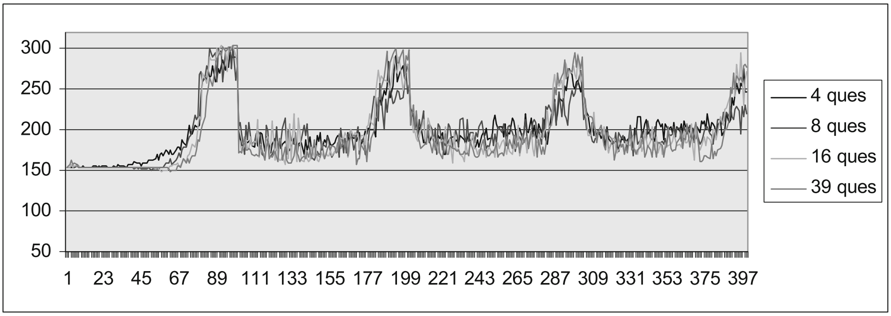

Fig. 9 Dynamics of average profit at industry level of dishonest agents with low direct experience-based knowledge

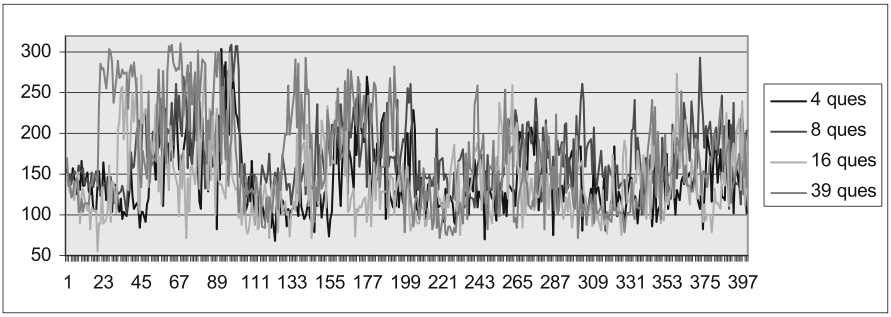

Fig. 10 Dynamics of average profit at industry level of dishonest agents with high direct experience-based knowledge

emphasized by the larger number of agents, as it happens when FTs play with their suppliers, and by the forgetfulness effects, which heavily impact only with dishonest agents. This can be seen clearly by comparing Figs. 6 and 10: despite the perfectly stable pattern of honest agents, with full cheaters performance grows until forgetfulness effects (partially) destroy agents' memory. Though such deletion concerns only inactivated suppliers and informers, it is enough to bring performance near the starting point (that corresponding to the random choice). And this cycle replicates periodically with the occurrence of forgetfulness mechanisms. Consequently, when this uncertainty applies to agents (clients) that engage many subcontractors at once, and moreover when potential agents are many, the impact of their errors are heavier, and thus, the dynamics becomes more chaotic.

5 Discussion and Conclusion

The following conclusions underline the main findings obtained from the present virtual experiments. Then they are compared with those of theoretical and empirical literature, and finally with those obtained with other simulation models. Five general conclusions can be drawn from this study. The first one is that opportunistic behaviours in the form of cheating severely damage industry competitiveness. The second one is that the exploration modes through direct and indirect experience have different effects on performance, and that its combination can have rather surprising outcomes, depending on structural and cognitive characteristics. In particular, at industry level a growth of internal knowledge does not improve performance, because it is heavily damaged by the forgetfulness mechanisms, which have a more accentuated impact on FTs. This happens especially with dishonest agents, because buyers spend an enormous time to control false information and to identify unreliable informers. Conversely, more external knowledge helps agents better and fast knowing the best suppliers. The size of the "mistake damage" is of primary importance for explaining the differentiated impact of external and internal knowledge growth on industry profitability. When it occurs through direct experience by engaging many subcontractors, clients' performance become very sensitive to mistakes, especially in a context of opportunistic behaviors.

The third main finding regards another important differentiation, namely that occurring between segments. The virtual experiments demonstrate that, even if they are submitted to the same rules and composed by almost identical size, they perform in a quite different way, and react in opposite direction to the growth of the two types of knowledge. Hence, the impact at the whole industry level depends on the combination of the two, and so what happens in real cases is quite unpredictable until a model does supply such disaggragated results. The major implication is that it cannot be taken for granted that an increase of a certain type of knowledge improves industry performance. This effect depends on the industry structure and on the exploration modes. This is a very original and interesting result, because current literature on the effects of knowledge growth and transfer on firms or industry profitability does not take into account this phenomenon.

There are two further implications of this analysis. The first one refers to the key-role played by FTs in a context of honest agents. If their competition increases, then their average quality keeps constant, while that of FPs grows significantly. Hence, to the extent that profit depends on quality, it means that industry quality competitiveness and profitability substantially improve. However, the side effect – eventually wishful, in a perspective of increasing industrial efficiency by pushing competition – is that the major part of FTs and a large part of FPs do not work any more in the long run. Conversely, if agents cheat, the same increasing of FTs competition implies a similar sacrifice in terms of firms' failures, which however is not compensated by any performance improvement at industry level.

The second implication is that the ratio of suppliers/buyers transactions is very important to determine the level and the sign of profitability in each segment. Such a ratio depends on the population size of buyers and suppliers and on both the number of suppliers per buyer (the outsourcing degree) and the number of clients per supplier (which here was kept fixed at 1). Further and specific research will say if the ratio of suppliers/buyers transactions is able to synthesize the effect of the competitive forces between suppliers and buyers (Porter, 1985) in all aspects or only for the issues investigated in this paper.

Finally, virtual experiments demonstrate that once the number of transactions exceeds that of suppliers, the marginal benefit of internal knowledge decreases, and uncertainty and instability sharply increase because of the high competitive pressure. Moreover, since the agents lack forms of advanced learning processes, performance improvements reached through direct experience are achieved at the price of higher instability. In such a context and with dishonest agents higher external knowledge produces further instability instead of performance improvements.

Despite the huge number of papers arguing that both knowledge growth and inter-firm non-opportunist behaviours increase collective performance at segment, industry or territory level, the empirical evidence is quite scarce and far from being conclusive. In particular, while there is a plenty of works concerning single firms' or industry competitive advantages due to patents, R&D expenses or other forms of possessed or accessed technological knowledge, there are no studies considering at once market knowledge in terms of information about competitors' reliability and suppliers' quality. In fact, there are few studies on the effects on single firms' or industry competitive advantages of these two variables taken separately, but not considered jointly. Moreover, they do not separate the effects of internal and external knowledge, which is the focus of this paper. Further, they consider forms of external knowledge much more reliable and structured than that analyzed in this work, because knowledge accessed through R&D or other forms of formal or informal agreements is more or less experienced directly. Thus, it could be considered as a type of internal knowledge.

Finally, in almost all these contributions opportunism is defined (more traditionally) as the betrayal of previous (formal or informal) trade agreements rather than in terms of giving false information, as it is done in this paper. Therefore, it is not possible to strictly confront the results of this model with the findings of empirical research or simulation models. In general terms, the present results confirm that opportunism damages industry profitability, as it was already argued by most authors. In comparing the supplier–buyer relationships in the automotive industry of Japan, US, and EU Sako (1998) found a positive relationship between trust and economic performance. His results have been further confirmed in a successive study by De Jong and Noteboom (2000).

The literature on supply chain is sharply divided between general-theoretical and operational works. In the modern approach to supply chain management partnering, integrating, collaborating and long-tern orientation are considered key factors of success (Tan et al., 1999). For its achievement trust (Doney and Cannon, 1997; Lee et al., 1997; Mason-Jones and Towill, 1998, 1999; Mohr and Speckman, 1994) and information exchange (Chen et al., 2000) are indicated as essential conditions. By collecting data from 125 North American manufacturing firms Zhou and Benton (2007) show that effective information sharing enhances effective supply chain practices.

Operational works recently employ simulation models extensively, but they are finalized to manage large and differentiated stocks in supply networks (Chatfield et al., 2006; Chen and Chen, 2009; Huang et al., 2005; Longo and Mirabelli, 2008) and to face with demand variability (Germain et al., 2008; Swafford et al., 2006). These models do not deal with the main problems addressed in this paper, and moreover they often overlook theoretical issues and non-technical market features.

However, when comparing the results of this work with those coming from other simulation models we can see quite strong consistency, although the originality of this analysis makes it quite incomparable regarding the main issues, that is the differentiated role played by internal and external knowledge growth. Besides this limit, a plenty of recent studies in supply-chain management underlines that low levels of trust can increase total supply chain costs considerably in promoting periods (Riddalls et al., 2002). Through a multi-agent simulation study Lin et al. (2005) find that trust mechanisms reduce the average cycle time, the in-time order fulfilment rate, and improve the selection of the best suppliers.

Of course, since this model derives from CIOPS, the highest comparability is between these two, even though the present model develops that one in a specific direction. In fact, though Biggiero and Sevi (2009) considers both internal and external knowledge, they do not analyze separately its effects, for instance when the growth of external knowledge is obtained by increasing only agents' absorptive capacity while keeping constant direct experience. Among the combinations analyzed in this work they consider only the default one, that is that with outsourcing degree of one subcontractor and absorptive capacity of four questions. Therefore, the comparison is limited to the fact that actually full dishonest behaviours severely damages industry profitability. It was implicitly suggested by most literature to expect that a growth of agents' knowledge base would have limited or even compensated the negative impact of opportunism. What this work adds more is the discovery of the sharp difference of the effects of increasing internal or external knowledge separately and in combination, and the evidence of the crucial role played by segments' performance.

With a quite similar model, Giardini et al., (2008) found as well that opportunism damages industry profitability. As Biggiero and Sevi (2009) they also confirm that, compared to a context with only direct experience, the use of indirect experience substantially increases industry profitability. However, they also do not deal with the effects of variations of internal and external knowledge. The negative impact of opportunism and cheating is confirmed also by Tykhonov et al., (2008) in a model of food supply chain with asymmetric information, and by Prietula (2001) in a model dealing with group effectiveness and efficiency. The comparability with the model of Klos and Noteboom (2001), which later on has been developed by Gorobets and Noteboom (2004), is high respect to the issue of supplier–buyer relationships and opportunism, but it is almost absent respect to the other issues, because agents are supposed to decide only according to direct experience and opportunism is intended only as betraying previous or current agreements.

The major limitations of this work concern parameters setting and architectural features of the model, because some of them increase and others decrease the stability or the level of profitability. Among the former there is the invariance of agents' behavior in terms of cheating, because if behavior could change agents' strategies would be subjected to much more uncertainty, and hence the whole industry would likely attain lower and later average profits. The model would be refined by introducing differentiated costs of the two modes of exploration: transaction costs associated to the various levels of outsourcing degree, and costs to acquire and manage information from others. Further, advanced forms of learning processes would certainly add interesting aspects to this analysis. Moreover, it is not implemented any strategy of building inter-firm networks, which also had to stabilize and maybe improve industry performance. Among the factors decreasing the stability or the level of profitability there is likely the introduction of mortality and natality mechanisms, which indeed would also substantially enrich the model, making it more useful for predictions and empirical validation, and would add room for more complex evolutionary patterns.

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Financial Fragility and Interacting Units: an Exercise

Carl Chiarella, Simone Giansante, Serena Sordi and Alessandro Vercelli

Abstract. This paper assumes that financial fluctuations are the result of the dynamic interaction between liquidity and solvency conditions of individual financial units. The framework is designed as a heterogeneous agent model which proceeds through discrete time steps within a finite time horizon. The interaction at the micro-level between financial units and the market maker, who is in charge of clearing the market, produces interesting complex dynamics. The model is analyzed by means of numerical simulations and agent-based computational economics (ACE) approach. The behaviour and evolution of financial units are studied for different parameter regimes in order to show the importance of the parameter setting in the emergence of complex dynamics. Monetary policy implications for the banking sector are also discussed.

1 Introduction

The complexity of economic cycles is the result of the impact of individual decisions on the macro variables of the economy. The interaction between the liquidity and solvency conditions of the individual financial units seems to play a crucial role in the emergence of complex financial fluctuations. The aim of the exercise presented

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here is to extend the financial fragility analysis initiated by Vercelli (2000) and further developed by Sordi & Vercelli (2006) and Dieci, Sordi & Vercelli (2006).

This work aims to provide micro-economic foundations to the model mentioned above. The framework is designed as a heterogeneous agent model which proceeds through discrete time steps within a finite time horizon. Financial units decide how many shares to buy or sell on the basis of supply and demand curves (linear and hyperbolic respectively). A market maker clears the market by aggregating the individual financial positions and taking an offsetting position. It also computes the price for the next period by means of a simple rule based on excess demand. Once the price is announced, financial units compute their current and intertemporal financial ratios to assess their liquidity and solvency conditions on the basis of their individual perceptions of the expected inflation rate and their financial fragility parameters. Financial units can also borrow money from the banking sector at the nominal interest rate announced at every time step by the Central Bank. More precisely, the nominal interest rate is determined by maintaining a relationship of equality between the total borrowing requirements of financial units, and the total supply of loans provided to the economy by the banking sector.

The paper is organized as follows. Section 2 presents the microfoundations of the framework by modelling the behaviour of financial units, and the reaction functions of the market maker and the Central Bank. Section 3 presents two exercises in which the model is capable of mimicking both stable and unstable equilibria as the consequence of individual behaviour. Section 4 concludes. Some mathematical results are contained in the Appendix.

2 The Model

The model proceeds through discrete timesteps. A set of N financial units (each labelled by a roman index $i = 1, \ldots, N$) interacts at each timestep t.

2.1 Price Dynamics

Financial units are heterogeneous in terms of their exchange strategies for the only risky asset in the economy. They decide how many shares to sell or to buy on the basis of supply and demand curves given by

$$p = f(q^S), \tag{1}$$

and

$$p = f(q^D), \tag{2}$$

respectively.

Accordingly, when, at time t, the market maker announces the price p_t, financial units choose their supplies, q_t^S, and demands, q_t^D, so as to satisfy (1) and (2), as shown in Fig. 1. Assuming, for simplicity, linear supply curves and hyperbolic demand curves for all N units, we can represent them according to the relations[1]

$$p_t = \alpha_{i,t} q_{i,t}^S, \tag{3}$$

and

$$p_t = \frac{\beta_{i,t}}{q_{i,t}^D}, \tag{4}$$

which, in terms of quantities may be written as

$$q_{i,t}^S = \frac{p_t}{\alpha_{i,t}}, \tag{5}$$

and

$$q_{i,t}^D = \frac{\beta_{i,t}}{p_t}. \tag{6}$$

Thus, the individual current inflow (y_i) and outflow (e_i) at time t are given by

$$y_{i,t} = p_t q_{i,t}^S = \frac{p_t^2}{\alpha_{i,t}}, \tag{7}$$

and

$$e_{i,t} = p_t q_{i,t}^D = \beta_{i,t}, \tag{8}$$

respectively. Therefore, the current realized financial ratio of the financial unit i $(c_{i,t})$ at time t is given by

$$c_{i,t} = \frac{e_{i,t}}{y_{i,t}} = \frac{\alpha_{i,t}\beta_{i,t}}{p_t^2}. \tag{9}$$

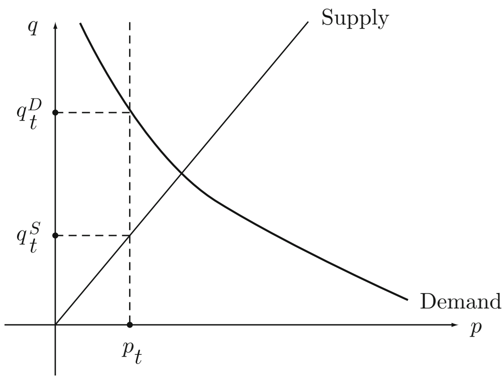


Fig. 1 Announcement of price p_t by the market-maker determines quantity demanded q_t^D and quantity supplied q_t^S, in period t by a typical agent

[1] We assume that agents determine their demand and supply schedules by solving an optimisation problem subject to their budget constraints. As the budget will evolve over time, the coefficients $\alpha_{i,t}$ and $\beta_{i,t}$ (dependent on it) will also be vary with time.

The individual demand and supply quantities of the N units are aggregated according to

$$D_t = \sum_{i=1}^{N} q_{i,t}^{D}, \tag{10}$$

and

$$S_t = \sum_{i=1}^{N} q_{i,t}^{S}. \tag{11}$$

Once D_t and S_t have been computed, the market maker clears the market by taking an offsetting long or short position and computes the price for the next period by using the rule

$$p_{t+1} = p_t \left[1 + \lambda_{\mathrm{M}} \left(\frac{D_t - S_t}{D_t} \right) \right] \tag{12}$$

where λ_{M} is the market maker's speed of adjustment of the price to excess demand. The adjustment rule (12) shows the market maker adjusting the price in reaction to the (relative) excess aggregate demand.

2.1.1 Desired Financial Ratio, Intertemporal Financial Ratio and Inflation Rate

Each financial unit i, at each time t, computes its *desired current* financial ratio $(k_{i,t})$, on the basis of the recursive relationship:

$$k_{i,t+1} = \max \left\{ k_{i,t} - \lambda_{k_i} \left[k_{i,t}^{*} - \left(1 - \mu^{i}\right) \right], 0 \right\}, \tag{13}$$

where the *intertemporal* financial ratio of unit i $(k_{i,t}^{*})$ at time t is given by

$$k_{i,t}^{*} = \frac{\sum_{s=0}^{T_i} \mathbb{E}_t^{(i)} \left[e_{i,t+s} \right] / \left(1 + \rho_{i,t}\right)^{s}}{\sum_{s=0}^{T_i} \mathbb{E}_t^{(i)} \left[y_{i,t+s} \right] / \left(1 + \rho_{i,t}\right)^{s}}, \tag{14}$$

with $\rho_{i,t}$ being the discount factor for the financial unit i at time t and T_i is its time horizon. We use $\mathbb{E}_t^{(i)}$ to denote expectations formed by unit i at time t. The expectations about inflows and outflows in future periods vary through time according to the following formula (see the Appendix for derivation):

$$k_{i,t}^{*} = c_{i,t} \left[\frac{(1 + r_t)^{T_i + 1} - 1}{r_t} \right] \left[\frac{r_t - \pi_{t,t+1}^{i}}{(1 + r_t)^{T_i + 1} - \left(1 + \pi_{t,t+1}^{i}\right)^{T_i + 1}} \right]. \tag{15}$$

After the market maker announces p_{t+1}, each financial unit updates its expected inflation for the next period according to the simple adaptive rule

$$\begin{aligned} \pi_{t,t+1}^{i} &= \pi_{t-1,t}^{i} + \lambda_{\pi}^{i} \left(\frac{p_t - p_{t-1}}{p_{t-1}} - \pi_{t-1,t}^{i} \right) \\ &= \lambda_{\pi}^{i} \left(\frac{p_t - p_{t-1}}{p_{t-1}} \right) + \left(1 - \lambda_{\pi}^{i}\right) \pi_{t-1,t}^{i}. \end{aligned}$$

Let $B_{i,t}$ be the budget constraint of financial unit i at time t, $z_{i,t}$ the amount borrowed in period t and $A_{i,t}$ the accumulated debt position at time t.

The budget constraint evolves according to

$$B_{i,t+1} = \max(B_{i,t} + (y_{i,t} - e_{i,t}) - r_t A_{i,t}, 0). \tag{16}$$

To determine the amount of borrowing desired in the next period the unit considers the difference between the outflow and inflow in the current period, adjusts this by the desired financial ratio and subtracts from this the budget available in period $t+1$. Thus the borrowing in period $t+1$ is given by

$$z_{i,t+1} = \left(e_{i,t}k_{i,t+1} - \frac{y_{i,t}}{k_{i,t+1}}\right) - B_{i,t+1}, \tag{17}$$

which upon use of (7) and (8) becomes

$$z_{i,t+1} = \left(\beta_{i,t}k_{i,t+1} - \frac{p_t^2}{\alpha_{i,t}k_{i,t+1}}\right) - B_{i,t+1}. \tag{18}$$

If the *desired* inflows and outflows in (18) are to equal the *actual* inflows and outflows in period $(t+1)$ then we must have

$$\beta_{i,t+1} = k_{i,t+1}\beta_{i,t}, \tag{19}$$

and

$$\alpha_{i,t+1} = k_{i,t+1}\alpha_{i,t}\left(\frac{p_{t+1}}{p_t}\right)^2. \tag{20}$$

Thus the accumulated debt position of the unit evolves according to

$$A_{i,t+1} = (1 + r_t)A_{i,t} + z_{i,t+1}. \tag{21}$$

Summing over all financial units we obtain the total borrowing in period $t+1$, namely

$$z_{t+1} = \sum_{i=1}^{N} z_{i,t+1} = \sum_{i=1}^{N}\left(\beta_{i,t}k_{i,t+1} - \frac{p_t^2}{\alpha_{i,t}k_{i,t+1}} - B_{i,t}\right) = \sum_{i=1}^{N}\left(\beta_{i,t+1} - \frac{p_{t+1}^2}{\alpha_{i,t+1}}\right). \tag{22}$$

Note that z_{t+1} will depend on r_t through its impact on $k_{i,t}^*$ and hence on $k_{i,t+1}$.

Let L_t be the total supply of loans provided to the economy by the banking sector. For the moment we assume the policy for L_t is set via Central Bank policies.

The interest rate r_t will be determined by the equality of z_t and L_t, that is

$$z_t = L_t, \tag{23}$$

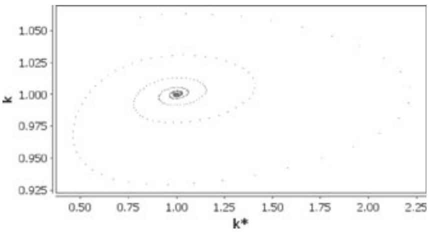
will implicitly determine r_t. However the explicit relation between z_t and r_t is very difficult to calculate. It may be simpler to assume that the Central Bank sets the interest rate according to the rule

$$r_{t+1} - r_t = \lambda_r\left(\frac{z_t - L_t}{z_t}\right), \tag{24}$$

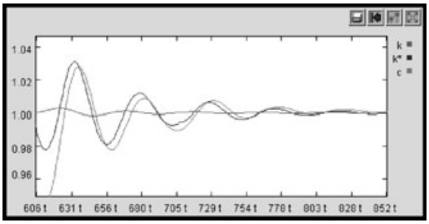
with $\lambda_r > 0$. Of course as $\lambda_r \to \infty$ we would obtain (23).

3 Simulation Results: Endogenous Fluctuations and Bankrupcy

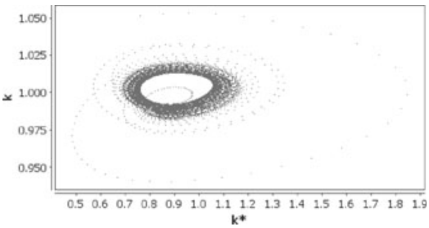
We now explore the dynamics of the model under different parameter settings in order to mimic both stable endogenous fluctuations and bankruptcy scenarios. Even if the model is capable of dealing with heterogeneous behaviours, for the sake of simplicity we restrict the analysis in this work to the interaction between a set of homogeneous financial units, the market maker and the private banking sector that provides liquidity to the units at the price r, which is set up at every step by the Central Bank[2]. In particular, we focus the analysis on two crucial parameters of individual behaviour, namely, the financial fragility threshold $1-\mu$ and the speed of adjustment of expected inflation λ_π.



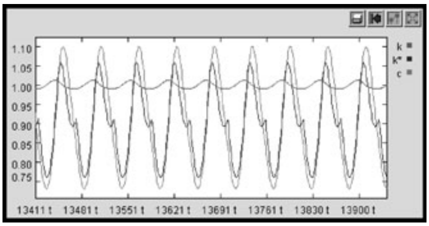
(a) $1-\mu=1, \lambda_\pi=0.06$



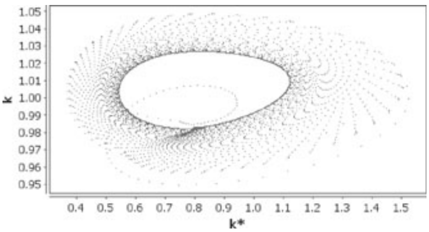
(b) $1-\mu=1, \lambda_\pi=0.06$



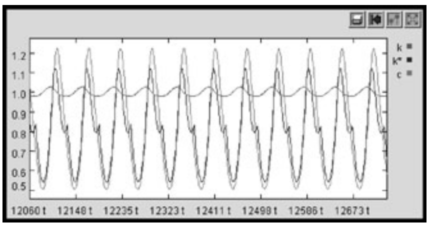
(c) $1-\mu=0.9, \lambda_\pi=0.06$



(d) $1-\mu=0.9, \lambda_\pi=0.06$



(e) $1-\mu=0.8, \lambda_\pi=0.06$



(f) $1-\mu=0.8, \lambda_\pi=0.06$

Fig. 2 Qualitative effect of decreasing the financial fragility threshold $1-\mu$

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[2] This simplification is carried out in order to facilitate calibration of the parameters. A more incisive analysis of the dynamics of interacting heterogeneous financial units will be carried out in a further work.

The remaining parameters are initialized as follows:

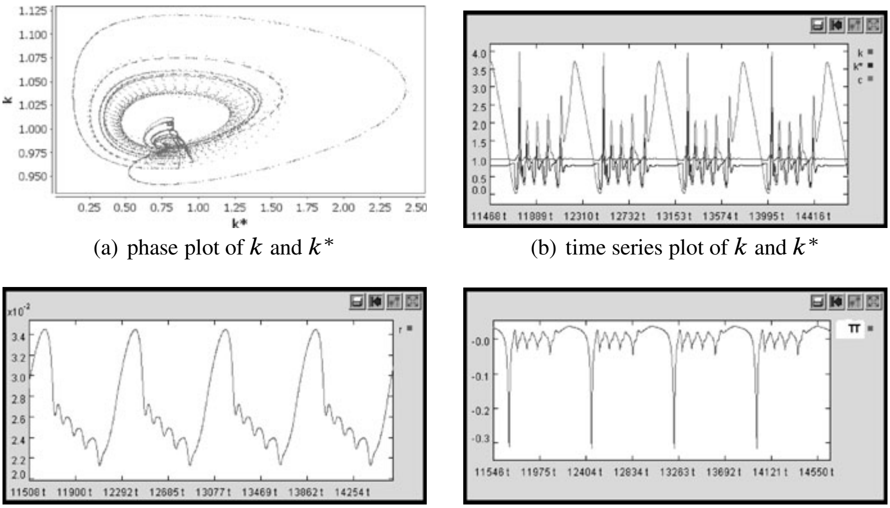
$$\lambda_r = 0.00005,\ L = 0,\ \lambda_k = 0.01$$

and simulations are initialized with

$$p_0 = 1,\ r_0 = 0.05,\ \pi_0 = 0.05,\ q_0^D = 1.2 \text{ and } q_0^S = 0.8.$$

We start the analysis with relatively low levels of the parameters λ_π and μ. Indeed, the parameter λ_π is set at the level $\lambda_\pi = 0.06$ and the financial fragility threshold varies in the range $0.8 \leq 1 - \mu \leq 1$.

Interesting results derive from simulations where financial units operate with the maximum financial threshold $1 - \mu = 1$. It represents the case in which very risk prone units compete with the market maker in driving the market price to very low levels and produce a locally asymptotically stable steady state (Fig. 2(a) and Fig. 2(b)). As μ increases, a stable closed curve exists (Fig. 2(c) and Fig. 2(d)) and the amplitude of the fluctuation becomes wider (Fig. 2(e) and Fig. 2(f)). The model exhibits more stable equilibria when financial units are more risk prone, which is a quite counterintuitive result in terms of real financial systems. This is due to the lack of any contagion effect among units and it represents an important limitation of this simplified version of the model. Indeed, the insolvency of highly exposed units should affect other units and bring more instability to the system by spreading losses around the financial network. However, results of this first exercise show a higher bargaining power of very exposed financial units.



(a) phase plot of k and k^* (b) time series plot of k and k^*

(c) time series plot of interest rate r (d) time series plot of expected inflation π

Fig. 3 Dynamics of high reaction level of inflation $\lambda_\pi = 0.1, 1 - \mu = 0.8$

A second run of tests deals with simulations where the speed of adjustment of expected inflation λ_π is set up beyond its safe level as reported in the previous section.

Figure 3 shows an example where financial units react strongly to changes in market price ($\lambda_\pi = 0.1$). They overreact on price movements taking more and more exposed positions (Fig. 3(d)). As a consequence, financial units are likely to go bankrupt (level of k^* much beyond 1 in Fig. 3(a) and Fig. 3(b)), forcing the Central Bank to intervene by increasing the interest rate r (Fig. 3(c)). Note that bankruptcy events happen on a regular basis along the business cycle, which could give interesting insights to policy makers.

4 Conclusions

The financial fragility model presented in this paper extends the analysis of financial fluctuations initiated by Vercelli (2000) and further developed by Sordi & Vercelli (2006) and Dieci et al. (2006) where the driving factor of financial fluctuations is the feedback between current and intertemporal financial ratios. The model investigates the market microfoundations of the system by modelling the behaviour of a market maker (whose actions provide a market clearing mechanism) and a private banking sector (whose actions provide liquidity to the financial units at an endogenous interest rate controlled by the Central Bank). Following analysis of the dynamic fluctuations reported in Dieci et al. (2006), the model is able to produce an interesting range of dynamic scenarios according to the value of the parameters. Like the results reported in Dieci et al. (2006), regular convergence to a steady state and regular fluctuations on attracting closed curves can be found in scenarios where financial units react slowly to price movements. As the reactivity of financial units increases, the market becomes more unstable driving units to bankruptcy. The intervention of the Central Bank in increasing the interest rate as a consequence of the rebalance of exposure in the money market is crucial to the restoration of stability to the market.

The results also show the importance of modelling mechanisms of financial contagion of virtually bankrupt units through their financial networks. This is a missing component in the version of the model presented here. It will be the focus of further investigation within the framework of this research programme.

Appendix

When units come to form expectations about inflows and outflows in future periods we assume that they expect these to grow by the accumulation of their expectation of inflation over the next period. Thus unit i's expectation of its outflow s periods ahead is given by

$$\mathbb{E}_t^{(i)}\left[e_{i,t+s}\right] = \mathbb{E}_t^{(i)}\left[\beta_{i,t+s}\right] = \beta_{i,t}\left(1 + \pi_{t,t+1}^{i}\right)^{s},$$

where $\pi^i_{t,t+1}$ is unit i's expected inflation rate from t to $t+1$. We can then rewrite the expected total outflow in the numerator of (14) as

$$E_{i,t} = \sum_{s=0}^{T_i} \frac{\mathbb{E}_t^{(i)}\left[e_{i,t+s}\right]}{(1+\rho_{i,t})^s} = \beta_{i,t} \sum_{s=0}^{T_i} \left(\frac{1+\pi^i_{t,t+1}}{1+\rho_{i,t}} \right)^s . \tag{25}$$

We assume that for each unit the relation between discount factors, nominal interest rate r_t and the individual expected inflation rate from t to $t+1$ has the form

$$1+\rho_{i,t} = (1+r_t)\left(1+\pi^i_{t,t+1}\right),$$

from which

$$\frac{1+\pi^i_{t,t+1}}{1+\rho_{i,t}} = \frac{1}{1+r_t}. \tag{26}$$

Thus, inserting the last equation into (25) we obtain

$$\begin{aligned} E_{i,t} &= \sum_{s=0}^{T_i} \frac{\mathbb{E}_t^{(i)}\left[e_{i,t+s}\right]}{(1+\rho_{i,t})^s} = \beta_{i,t} \sum_{s=0}^{T_i} \left(\frac{1}{1+r_t} \right)^s \\ &= \beta_{i,t} \left[\frac{1-(1+r_t)^{-(T_i+1)}}{1-(1+r_t)^{-1}} \right] = \frac{\beta_{i,t}\left[(1+r_t)^{T_i+1}-1\right]}{r_t(1+r_t)^{T_i}}. \end{aligned} \tag{27}$$

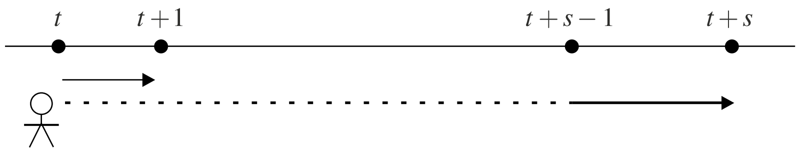
With regard to the denominator of (14), we can similarly compute

$$\mathbb{E}_t^{(i)}\left[y_{i,t+s}\right] = \frac{1}{\alpha_{i,t}} \mathbb{E}_t^{(i)}\left[p^2_{t+s}\right] \tag{28}$$

$$= \frac{1}{\alpha_{i,t}} p_t^2 \left(1+\pi^i_{t|t,t+1}\right)^2 \left(1+\pi^i_{t|t+1,t+2}\right)^2 \cdots \left(1+\pi^i_{t|t+s-1,t+s}\right)^2, \tag{29}$$

where (see Fig. 4)

$$\pi^i_{t|t+s-1,t+s} = \mathbb{E}_t^{(i)}\left[\frac{p_{t+s}-p_{t+s-1}}{p_{t+s-1}}\right] = \begin{cases} \text{the expectation formed by} \\ \text{unit } i \text{ at time } t \text{ of inflation} \\ \text{over the period} \quad (t+s-1, t+s) \end{cases}$$



$$\mathbb{E}_t^{(i)}\left[\frac{p_{t+s-1}-p_{t+s}}{p_{t+s-1}}\right] \equiv \pi_{t|t+s-1,t+s}$$

Fig. 4 Agent i assumes $\pi_{t|t+s-1,t+s} = \pi^i_{t|t,t+1} \equiv \pi^i_{t,t+1}$

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Part III
Techniques and Tools

Using Homogeneous Groupings in Portfolio Management

Jaime Gil-Aluja, Anna M. Gil-Lafuente and Jaime Gil-Lafuente

Abstract. Often, in situations of uncertainty in portfolio management, it is difficult to apply the numerical methods based on the linearity principle. When this happens it is possible to use nonnumeric techniques to assess the situations with a non linear attitude. One of the concepts that can be used in these situations is the concept of grouping.

In the last thirty years, several studies have tried to give good solutions to the problems of homogeneous groupings. For example, we could mention the Pichat algorithm, the affinities algorithms and several studies developed by the authors of this work.

In this paper, we use some topological axioms in order to develop an algorithm that is able to reduce the number of elements of the power sets of the related sets by connecting them to the sets that form the topologies. We will apply this algorithm in the grouping of titles listed in the Stock Exchange or in its dual perspective.

1 The Two Perspectives for Topological Fuzzification in Economy

It is well known that a topology E in uncertainty can be defined by the subset $T(E)$ of the opened that accomplishes the following axioms (Chang, 1968). Note that for further reading on fuzzy topology and pretopology, we recommend, for example (Badard, 1981; Bayoumi, 2005; Du et. al., 2005; Fang and Chen, 2007; Fang and

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Yue, 2004; Gil-Aluja, 2003; Gil-Aluja and Gil-Lafuente, 2007; Saadati and Park, 2006; Yan and Wu, 2007; Yue, 2007):

1. $\varnothing \in T(E)$
2. $E \in T(E)$
3. $(\underset{\sim}{A}_j \in T(E), \underset{\sim}{A}_k \in T(E)) \to (\underset{\sim}{A}_j \cap \underset{\sim}{A}_k \in T(E))$
4. $(\underset{\sim}{A}_j \in T(E), \underset{\sim}{A}_k \in T(E)) \to (\underset{\sim}{A}_j \cup \underset{\sim}{A}_k \in T(E))$

where $\underset{\sim}{A}_j$, $\underset{\sim}{A}_k$ may have a different meaning depending on the criteria used for the fuzzification. In the first case, they are fuzzy subsets of the referential set E that accomplishes the previous axioms, and in the other case, they are elements of the power set established from a referential set E of fuzzy subsets. In the first case, the referential set E is formed initially by the elements of the referential set of the fuzzy subsets. In the second case, their elements are the fuzzy subsets theirselves. As it has been pointed out in other works (Gil-Aluja, 2003; Gil-Aluja and Gil-Lafuente, 2007) the selection of one of these perspectives depends mainly on the objectives of the analysis.

In an economical and financial context, we consider that it is relevant to think about the meaning of the components of both cases. For doing this, we will use the representability of the notion of fuzzy subset. The reason is because for an economist, a fuzzy subset is a descriptor of a physical or mental object; and this description is developed by putting different levels to the elements of the referential set formed by the attributes of the objects that we want to describe. Then, in the economic environment it is possible to accept that in the first case, the referential set E is formed by the set of attributes that describe each object while in the second case, the referential set E is formed by the fuzzy subsets, where each of them describe an object.

If we consider financial products such as titles listed in the Stock Exchange, the description of each of them will take place by a certain number of attributes such as the expected rentability, the liquidity capacity without loses, etc., all of them classified at certain level. In this assumption, the referential set E will be formed in the first case by the expected rentability, the liquidity capacity, etc., and in the second case, by the different titles listed in the Stock Exchange.

With this approach, the $\underset{\sim}{A}_j, \underset{\sim}{A}_k \in T(E)$, the elements of the open set $T(E)$, are in the first case, fuzzy subsets with the referential of their attributes and in the second case, fuzzy subsets or groupings of fuzzy subsets with the same referential.

It is obvious that the concept of economic representability is different in each case. Then, the Axioms 1 and 2 acquire the following meaning:

- In the first case, Axiom 1 shows that the fuzzy subset (title listed in the Stock Exchange) with a null level in all its attributes is an open set and so is (Axiom 2) the fuzzy subset (title listed in the Stock Exchange) with level one (maximum) in all its attributes.
- In the second case, Axiom 1 shows that in a situation without fuzzy subsets we have an open set. In this case, the set of all the fuzzy sets (all the titles listed in the Stock Exchange) is also an open set (Axiom 2).

In Axiom 3, we also find different meanings depending on the case analyzed:

- In the first one, Axiom 3 requires that if a fuzzy subset with certain levels for each attribute is an open set and so is another fuzzy subset with its own levels, then, there exists a third one that it is also an open set with a membership level for each attribute that is equal to the lowest of the other two.
- In the second one, we can see that if a group of fuzzy subsets is an open set and so is another group of fuzzy subsets, then, the group of fuzzy subsets that is contained in both groups, is also an open set.

Finally, Axiom 4 expresses the following for each case:

- In the first one, if we have a fuzzy subset with certain levels for each attribute and another one with its own levels, and both are open sets, then, there exists another fuzzy subset that it is also an open set. The membership level of the attributes of this fuzzy subset is given by the maximum between the other two fuzzy subsets.
- In the second one, if we have two groups of fuzzy subsets that are open sets, then, there exists a third one that is also an open set and it comprises the fuzzy subsets of the first and/or the second group.

Sometimes, it can be useful to use as open sets, the complementary of any open set. This implies the necessity of considering another axiom as follows:

5. $(\underset{\sim}{A}_j \in T(E)) \rightarrow (\underset{\sim}{A}_j \in T(E))$

In this axiom, the representativity also acquires a different meaning depending on the case used. Then:

- In the first case, it is necessary that if a fuzzy set is an open set, then, the fuzzy subset which has a complimentary level to the first one in all the attributes has also to be an open set. Then, if for a certain attribute an open set has a level α the complimentary fuzzy subset will have $1 - \alpha$, where $\alpha \in [0, 1]$.
- In the second case, when a grouping of fuzzy subsets is an open set, then, the group formed by the rest of fuzzy subsets is also an open set.

Focusing in this important context, we believe that it is interesting to note that it is not necessary to establish the existence of the five axioms presented above for arriving to the same result. This happens because if three of the axioms are accomplished, then, the other two will be accomplished automatically. These three axioms are:

1. $E \in T(E)$
2. $(\underset{\sim}{A}_j \in T(E)) \rightarrow (\underset{\sim}{A}_j \in T(E))$
3. $(\underset{\sim}{A}_j \in T(E), \underset{\sim}{A}_k \in T(E)) \rightarrow (\underset{\sim}{A}_j \cup \underset{\sim}{A}_k \in T(E))$

As we can see, with the first and the second axiom, it is satisfied: $\varnothing \in T(E)$

And, due to: $\underset{\sim}{A}_j \cup \underset{\sim}{A}_k \in T(E)$

Is also: $\overline{\underset{\sim}{A}_j \cup \underset{\sim}{A}_k} \in T(E)$

By using De Morgan theorem: $\overline{\underset{\sim}{A}_j \cup \underset{\sim}{A}_k} = \overline{\underset{\sim}{A}_j} \cap \overline{\underset{\sim}{A}_k}$

Then: $\overline{\underset{\sim}{A}_j} \cap \overline{\underset{\sim}{A}_k} \in T(E)$

Now, it is interesting to establish these two general cases in the financial environment and its representativity in a real problem of the financial operations. Note that in this paper we will only focus on the first case.

2 The Hypothesis of a Referential Set of Referentials

We will assume a set of attributes of titles listed in the Stock Exchange that are significant for the potential investors.

Assume a referential set E of attributes of titles listed in the Stock Exchange, as follows:

$$E = \{x_j / j = 1, 2, \ldots, m\}$$

where the x_j represent, for example, the expected rentability, the liquidity capacity, etc.

Now, we describe each title by using a fuzzy subset of the referential of its attributes that we designate as A_k, $k = 1, 2, \ldots, n$, where n indicates the total number of titles considered. Then, each of these titles will be described as follows:

$$\left(E, \mu \underset{\sim}{A}_k(x_j)\right), \quad \mu \underset{\sim}{A}_k \in [0, 1]$$

Then, we establish a relation between title and attribute, such that if $x_j \in E$ possess a value of the membership function for $\underset{\sim}{A}_k$ with a level μ, we write it as:

$$\mu \underset{\sim}{A}_k(x_j) = \mu, \quad \mu \in [0, 1]$$

Lets see an example. Assume 5 attributes:

x_1 = expected rentability.
x_2 = liquity level.
x_3 = sustainability in the quotation.
x_4 = appearance of a public offering.
x_5 = prestige of the quoted society.

And four titles that can be described as follows:

	x_1	x_2	x_3	x_4	x_5
$\underset{\sim}{A}_1 =$	.9	.7	.8	.4	.7

	x_1	x_2	x_3	x_4	x_5
$\underset{\sim}{A}_2 =$	.8	.5	.9	.9	1

	x_1	x_2	x_3	x_4	x_5
$\underset{\sim}{A}_3 =$	.4	.9	.9	.7	.8

	x_1	x_2	x_3	x_4	x_5
$\underset{\sim}{A}_4 =$	.6	.8	.9	.8	.9

The description of these titles by its attributes permit us to know the expected level of each attribute $x_j / j = 1, 2, 3, 4$ that has been assigned for each title.

However, the investor of the titles often establishes, for each attribute, a minimum level or threshold, where he assumes that a level below the threshold can be considered as zero. Then, it is necessary to establish a fuzzy subset of thresholds that we will designate as U where:

$$\mu_U(x_j) = \lambda_j \in [0, 1]$$

This means that we establish α_j-cuts such that:

$$< \alpha_j \to 0$$
$$\geqslant \alpha_j \to 1$$

We continue the example creating the fuzzy subset of thresholds:

	x_1	x_2	x_3	x_4	x_5
$U =$	.8	.7	.8	.7	.8

Then, the fuzzy subsets $A_k, k = 1, 2, 3, 4$ become the following Boolean subsets:

	x_1	x_2	x_3	x_4	x_5	
$A_1^{(\alpha)} =$	1	1	1			$= \{x_1, x_2, x_3\}$
$A_2^{(\alpha)} =$	1		1	1	1	$= \{x_1, x_3, x_4, x_5\}$
$A_3^{(\alpha)} =$	1	1	1	1	1	$= \{x_2, x_3, x_4, x_5\}$
$A_4^{(\alpha)} =$		1	1	1	1	$= \{x_2, x_3, x_4, x_5\}$

Now, we are able to form the family F that comprises the group of attributes that are possessed, at certain level, by the four titles:

$$F = \{\{x_1, x_2, x_3\}, \{x_1, x_3, x_4, x_5\}, \{x_2, x_3, x_4, x_5\}, \{x_2, x_3, x_4, x_5\}\}$$

Note that a key aspect in this process is the assignment of valuations to the subset of thresholds, because depending on these valuations, the family F will be different.

In the following, we present each of the elements that form the family and also its complementaries:

$$F\{A_1^{(\alpha)}\} = \{x_1, x_2, x_3\} \qquad \bar{F}\{A_1^{(\alpha)}\} = \{x_4, x_5\}$$
$$F\{A_2^{(\alpha)}\} = \{x_1, x_3, x_4, x_5\} \qquad \bar{F}\{A_2^{(\alpha)}\} = \{x_2\}$$
$$F\{A_3^{(\alpha)}\} = \{x_2, x_3, x_4, x_5\} \qquad \bar{F}\{A_3^{(\alpha)}\} = \{x_1\}$$
$$F\{A_4^{(\alpha)}\} = \{x_2, x_3, x_4, x_5\} \qquad \bar{F}\{A_4^{(\alpha)}\} = \{x_1\}$$

With this, we have shown in the first column the attributes possessed by each title and in the second column the attributes that are not possessed by them.

For simplifying the notation, we will assume:

$$F\left(A_1^{(\alpha)}\right) = F_1\,,\ \ F\left(A_2^{(\alpha)}\right) = F_2\,,\ \ F\left(A_3^{(\alpha)}\right) = F\left(A_4^{(\alpha)}\right) = F_{3,4}$$

Next, we analyze the attributes in order to know if one or more of them are possessed by all the titles or not. This analysis can be done by using all the available intersections between F_i and $\bar{F}_i$. Then:

$$\begin{aligned} &F_1 \cap F_2 \cap F_{3,4} = \{x_3\} && F_1 \cap F_2 \cap F_{3,4} = \{\bar{x}_1\} \\ &F_1 \cap \bar{F}_2 \cap F_{3,4} = \{x_2\} && \bar{F}_1 \cap F_2 \cap F_{3,4} = \{x_4, x_5\} \\ &F_1 \cap \bar{F}_2 \cap \bar{F}_{3,4} = \varnothing && \bar{F}_1 \cap F_2 \cap \bar{F}_{3,4} = \varnothing \\ &\bar{F}_1 \cap \bar{F}_2 \cap F_{3,4} = \varnothing && \bar{F}_1 \cap \bar{F}_2 \cap \bar{F}_{3,4} = \varnothing \end{aligned}$$

Then, we get the following nonnull intersections:

$$\{x_1\}, \{x_2\}, \{x_3\}, \{x_4, x_5\}$$

As we can see, the attributes can be separated, according to the number of titles that possess them. In this case, we have four groups where three of them are constituted only by one element and the last group by two elements.

By using the largest number of titles for each attribute, we get an optimization. Then, if we unify all these attributes in all the different ways and we add the empty set, we get:

$$\begin{aligned} T(E_2) =&\{\varnothing\{x_1\}, \{x_2\}, \{x_3\}, \{x_4, x_5\}, \{x_1, x_2\}, \{x_1, x_3\}, \{x_1, x_4, x_5\}, \{x_2, x_3\}, \\ &\{x_2, x_4, x_5\}, \{x_3, x_4, x_5\}, \{x_1, x_2, x_3\}, \{x_1, x_2, x_4, x_5\}, \{x_1, x_3, x_4, x_5\}, \\ &\{x_2, x_3, x_4, x_5\}, E_2\} \end{aligned}$$

that is a topology.

As it is well known, a topology can be represented by using a Boolean lattice that in our example is:

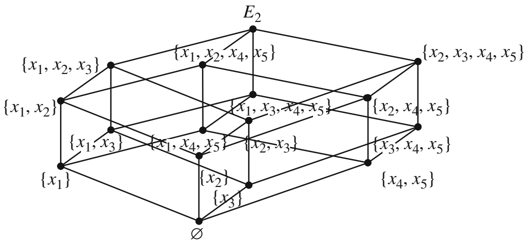

Fig. 1 A Boole lattice that represents a topology

It is clear that all the axioms commented above are accomplished. Note that this is obvious because it is a Boolean lattice.

3 The Dual Approach

Following with the same problem, it is interesting to consider the topology if instead of describing each of the tittles by using fuzzy subsets of the referencial of the attributes, we describe each attribute by using the levels they get in the titles. Then, in our example, we get the following, after having considered the threshold subset:

$$x_1 = \begin{array}{|c|c|c|c|} \multicolumn{1}{c}{A_1^{(\alpha)}} & \multicolumn{1}{c}{A_2^{(\alpha)}} & \multicolumn{1}{c}{A_3^{(\alpha)}} & \multicolumn{1}{c}{A_4^{(\alpha)}} \\ \hline 1 & 1 & & \\ \hline \end{array} = \{A_1^{(\alpha)}, A_2^{(\alpha)}\}$$

$$x_2 = \begin{array}{|c|c|c|c|} \multicolumn{1}{c}{A_1^{(\alpha)}} & \multicolumn{1}{c}{A_2^{(\alpha)}} & \multicolumn{1}{c}{A_3^{(\alpha)}} & \multicolumn{1}{c}{A_4^{(\alpha)}} \\ \hline 1 & & 1 & 1 \\ \hline \end{array} = \{A_1^{(\alpha)}, A_3^{(\alpha)}, A_4^{(\alpha)}\}$$

$$x_3 = \begin{array}{|c|c|c|c|} \multicolumn{1}{c}{A_1^{(\alpha)}} & \multicolumn{1}{c}{A_2^{(\alpha)}} & \multicolumn{1}{c}{A_3^{(\alpha)}} & \multicolumn{1}{c}{A_4^{(\alpha)}} \\ \hline 1 & 1 & 1 & 1 \\ \hline \end{array} = \{A_1^{(\alpha)}, A_2^{(\alpha)}, A_3^{(\alpha)}, A_4^{(\alpha)}\}$$

$$x_4 = \begin{array}{|c|c|c|c|} \multicolumn{1}{c}{A_1^{(\alpha)}} & \multicolumn{1}{c}{A_2^{(\alpha)}} & \multicolumn{1}{c}{A_3^{(\alpha)}} & \multicolumn{1}{c}{A_4^{(\alpha)}} \\ \hline & 1 & 1 & 1 \\ \hline \end{array} = \{A_2^{(\alpha)}, A_3^{(\alpha)}, A_4^{(\alpha)}\}$$

$$x_5 = \begin{array}{|c|c|c|c|} \multicolumn{1}{c}{A_1^{(\alpha)}} & \multicolumn{1}{c}{A_2^{(\alpha)}} & \multicolumn{1}{c}{A_3^{(\alpha)}} & \multicolumn{1}{c}{A_4^{(\alpha)}} \\ \hline & 1 & 1 & 1 \\ \hline \end{array} = \{A_2^{(\alpha)}, A_3^{(\alpha)}, A_4^{(\alpha)}\}$$

Following the same way than before, we find the family of titles, formed by the subset of the ones that possess the attribute x_1, the attribute x_2, etc., until the attribute x_5:

$$\begin{aligned} F = &\{\{A_1^{(\alpha)}, A_2^{(\alpha)}\}, \{A_1^{(\alpha)}, A_3^{(\alpha)}, A_4^{(\alpha)}\}, \{A_1^{(\alpha)}, A_2^{(\alpha)}, A_3^{(\alpha)}, A_4^{(\alpha)}\}, \{A_2^{(\alpha)}, A_3^{(\alpha)}, A_4^{(\alpha)}\}, \\ &\{A_2^{(\alpha)}, A_3^{(\alpha)}, A_4^{(\alpha)}\}\} \end{aligned}$$

Next, we present each of the elements of this new family F and its complimentaries:

$$\begin{aligned} F(x_1) &= \{A_1^{(\alpha)}, A_2^{(\alpha)}\} & F(x_1) &= \{\bar{A}_3^{(\alpha)}, A_4^{(\alpha)}\} \\ F(x_2) &= \{A_1^{(\alpha)}, A_3^{(\alpha)}, A_4^{(\alpha)}\} & F(x_2) &= \{\bar{A}_2^{(\alpha)}\} \\ F(x_3) &= \{A_1^{(\alpha)}, A_2^{(\alpha)}, A_3^{(\alpha)}, A_4^{(\alpha)}\} & F(x_3) &= \bar{\varnothing} \\ F(x_4) &= \{A_2^{(\alpha)}, A_3^{(\alpha)}, A_4^{(\alpha)}\} & F(x_4) &= \{\bar{A}_1^{(\alpha)}\} \\ F(x_5) &= \{A_2^{(\alpha)}, A_3^{(\alpha)}, A_4^{(\alpha)}\} & F(x_5) &= \{\bar{A}_1^{(\alpha)}\} \end{aligned}$$

The coincidence of two elements of the family, as it happens in this case, permits that we only need to consider one representative grouping of the fuzzy subsets that possess the attributes x_4 and x_5.

Now, we will develop all the possible intersections. In order to use an easier presentation, we will use $F_1 = F(x_1)$, $F_2 = F(x_2)$, $F_3 = F(x_3)$, $F_{4,5} = F(x_4) = F(x_5)$. We get the following:

$$F_1 \cap F_2 \cap F_3 \cap F_{4,5} = \varnothing\,, \qquad F_1 \cap F_2 \cap F_3 \cap \bar{F}_{4,5} = A_1^{(\alpha)}$$
$$F_1 \cap F_2 \cap \bar{F}_3 \cap F_{4,5} = \varnothing, \qquad F_1 \cap F_2 \cap \bar{F}_3 \cap F_{4,5} = A_2^{(\alpha)}$$
$$\bar{F}_1 \cap F_2 \cap F_3 \cap F_{4,5} = \{A_3^{(\alpha)}, A_4^{(\alpha)}\}, \qquad F_1 \cap F_2 \cap \bar{F}_3 \cap \bar{F}_{4,5} = \varnothing$$
$$F_1 \cap \bar{F}_2 \cap F_3 \cap \bar{F}_{4,5} = \varnothing, \qquad \bar{F}_1 \cap F_2 \cap F_3 \cap \bar{F}_{4,5} = \varnothing$$
$$F_1 \cap \bar{F}_2 \cap \bar{F}_3 \cap F_{4,5} = \varnothing\,, \qquad \bar{F}_1 \cap F_2 \cap \bar{F}_3 \cap F_{4,5} = \varnothing$$
$$\bar{F}_1 \cap \bar{F}_2 \cap F_3 \cap F_{4,5} = \varnothing\,, \qquad F_1 \cap \bar{F}_2 \cap \bar{F}_3 \cap \bar{F}_{4,5} = \varnothing$$
$$\bar{F}_1 \cap F_2 \cap \bar{F}_3 \cap \bar{F}_{4,5} = \varnothing\,, \qquad \bar{F}_1 \cap \bar{F}_2 \cap F_3 \cap \bar{F}_{4,5} = \varnothing$$
$$\bar{F}_1 \cap \bar{F}_2 \cap \bar{F}_3 \cap F_{4,5} = \varnothing\,, \qquad \bar{F}_1 \cap \bar{F}_2 \cap \bar{F}_3 \cap \bar{F}_{4,5} = \varnothing$$

The non empty intersections are:

$$\{A_1^{(\alpha)}\}, \{A_2^{(\alpha)}\}, \{A_3^{(\alpha)}, A_4^{(\alpha)}\}$$

We get the desired topology considering these elements and all their possible unions, and adding the empty set. Then:

$$T(E_1) = \Big\{\varnothing, \{A_1^{(\alpha)}\}, \{A_2^{(\alpha)}\}, \{A_3^{(\alpha)}, A_4^{(\alpha)}\}, \{A_1^{(\alpha)}, A_2^{(\alpha)}\}, \{A_1^{(\alpha)}, A_3^{(\alpha)}, A_4^{(\alpha)}\},$$
$$\{A_2^{(\alpha)}, A_3^{(\alpha)}, A_4^{(\alpha)}\}, E_1\Big\}$$

We can represent with a circle this topology inside a Boolean lattice of the power set:

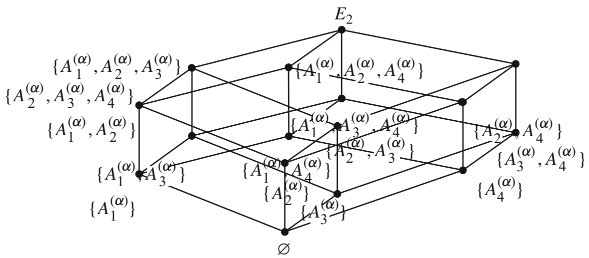

Fig. 2 A Boole lattice of the power set

As it is well known, a topology forms a Boolean lattice. In this example, it is as follows:

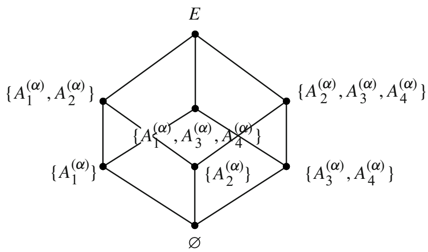

Fig. 3 A topology that forms a Boole lattice

In this case, it is also straightforward to prove the axiomatic requirements.

4 Relation Between the Two Topologies

Now, we go back to the beginning to present the information related to the descriptors of the titles, once developed the adjustment with thresholds. Note that now we will develop the analysis using the matrix form.

	x_1	x_2	x_3	x_4	x_5
$A_1^{(\alpha)}$	1	1	1		
$A_2^{(\alpha)}$	1		1	1	1
$A_3^{(\alpha)}$		1	1	1	1
$A_4^{(\alpha)}$		1	1	1	1

Next, we relate each of the elements of the topology $T(E_2)$ with those elements of the power set of E_1, the titles that all of them possess and the attributes that each element of the topology $T(E_2)$ establishes. In the following, we present the results obtained in the example:

$$
\begin{array}{lcl}
\varnothing & \rightarrow & E_1 \\
\{x_1\} & \rightarrow & \{A_1^{(\alpha)}, A_2^{(\alpha)}\} \\
\{x_2\} & \rightarrow & \{A_1^{(\alpha)}, A_3^{(\alpha)}, A_4^{(\alpha)}\} \\
\{x_3\} & \rightarrow & E_1 \\
\{x_4, x_5\} & \rightarrow & \{A_2^{(\alpha)}, A_3^{(\alpha)}, A_4^{(\alpha)}\} \\
\{x_1, x_2\} & \rightarrow & \{A_1^{(\alpha)}\} \\
\{x_1, x_3\} & \rightarrow & \{A_1^{(\alpha)}, A_2^{(\alpha)}\} \\
\{x_1, x_4, x_5\} & \rightarrow & \{A_2^{(\alpha)}\} \\
\{x_2, x_3\} & \rightarrow & \{A_1^{(\alpha)}, A_3^{(\alpha)}, A_4^{(\alpha)}\} \\
\{x_2, x_4, x_5\} & \rightarrow & \{A_3^{(\alpha)}, A_4^{(\alpha)}\} \\
\{x_3, x_4, x_5\} & \rightarrow & \{A_2^{(\alpha)}, A_3^{(\alpha)}, A_4^{(\alpha)}\} \\
\{x_1, x_2, x_3\} & \rightarrow & \{A_1^{(\alpha)}\} \\
\{x_1, x_2, x_4, x_5\} & \rightarrow & \varnothing \\
\{x_1, x_3, x_4, x_5\} & \rightarrow & \{A_2^{(\alpha)}\} \\
\{x_2, x_3, x_4, x_5\} & \rightarrow & \{A_3^{(\alpha)}, A_4^{(\alpha)}\} \\
E_2 & \rightarrow & \varnothing
\end{array}
$$

As we can see, all the groups of titles found are part of the topology $T(E_1)$.

Now, we will present the correspondences represented in the two lattices formed by the topologies $T(E_1)$ and $T(E_2)$:

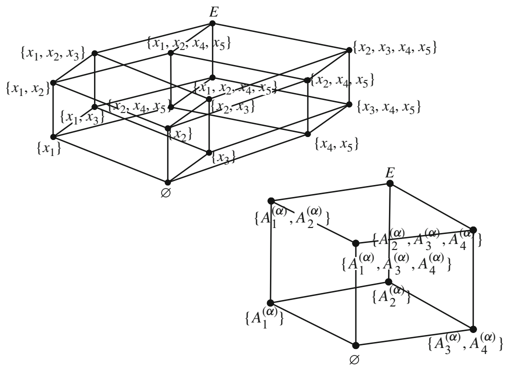

Fig. 4 Correspondences represented with two lattices

As we can see, in each vertex of the lattice corresponding to the topology $T(E_1)$ it arrives two lines that come from two vertexes of the lattice of the topology $T(E_2)$.

In the other hand, when it has been established the correspondence between the elements of the topology $T(E_2)$ it has been selected the group of elements of the power set of E_2, that is to say, the group with a larger number of attributes, excluding those groups formed by a smaller quantity. What it has been previously developed visually, now it can be found automatically by choosing in each vertex of the lattice of the topology $T(E_1)$, the line that conducts to the vertex of the lattice of the topology $T(E_2)$ with a larger number of elements, that is, attributes.

Perhaps, this can be seen more clearly with the following graph:

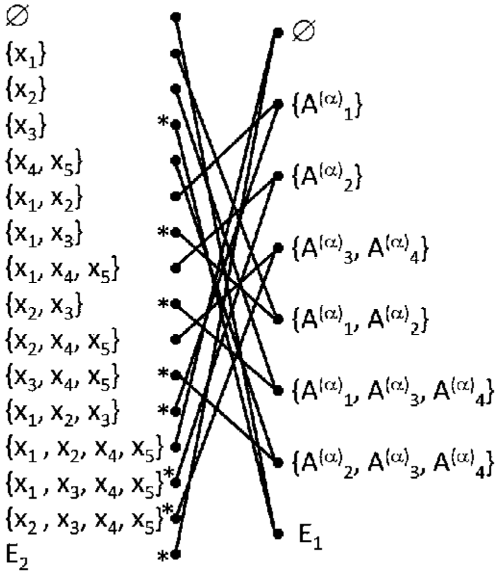

Fig. 5 Correspondence between the two topologies

From another perspective, if we consider all the elements of the topology $T(E_1)$, that is, titles and groups of titles, and the subsets of attributes possessed by all of them are found visually in the matrix, then, we get:

$$\begin{array}{lcl}
\varnothing & \rightarrow & E_2 \\
\{A_1^{(\alpha)}\} & \rightarrow & \{x_1, x_2, x_3\} \\
\{A_2^{(\alpha)}\} & \rightarrow & \{x_1, x_3, x_4, x_5\} \\
\{A_3^{(\alpha)}, A_4^{(\alpha)}\} & \rightarrow & \{x_2, x_3, x_4, x_5\} \\
\{A_1^{(\alpha)}, A_2^{(\alpha)}\} & \rightarrow & \{x_1, x_3\} \\
\{A_1^{(\alpha)}, A_3^{(\alpha)}, A_4^{(\alpha)}\} & \rightarrow & \{x_2, x_3\} \\
\{A_2^{(\alpha)}, A_3^{(\alpha)}, A_4^{(\alpha)}\} & \rightarrow & \{x_3, x_4, x_5\} \\
E_1 & \rightarrow & \{x_3\}
\end{array}$$

where the attribute x_3 and the groups of attributes found form part of the topology $T(E_2)$.

We observe that due to the fact that the topology $T(E_1)$ has a lower number of elements than the topology $T(E_2)$ the correspondence of each element of $T(E_1)$ goes to only one element of $T(E_2)$. This is interesting in order to develop an algorithm for groupings.

At least, we have shown that there exists a correspondence between both topologies. In the business and in the economic environment, it becomes interesting to analyze this type of correspondence, especially when we want to establish homogeneous groupings or segmentation processes.

Finally, lets see the groupings found by using a lattice:

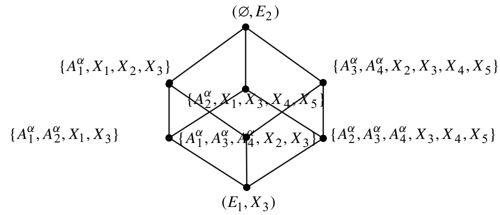

Fig. 6 Groupings by using a lattice

The same result is found by using one of the algorithms used for obtaining affinities (Kaufmann and Gil-Aluja, 1991). But then, it is necessary to consider, in the relation between the set of titles and the set of attributes, all the elements of the power set of both sets.

Then, starting from the same matrix:

	x_1	x_2	x_3	x_4	x_5
$A_1^{(\alpha)}$	1	1	1		
$A_2^{(\alpha)}$	1		1	1	1
$A_3^{(\alpha)}$		1	1	1	1
$A_4^{(\alpha)}$		1	1	1	1

and using the algorithm of the maximum inverse correspondence (Gil-Aluja, 1999) we find the right connection:

$$\begin{array}{llll}
\varnothing & \rightarrow & E_2 & \leftarrow \\
A_1^{(\alpha)} & \rightarrow & x_1, x_2, x_3 & \leftarrow \\
A_2^{(\alpha)} & \rightarrow & x_1, x_3, x_4, x_5 & \leftarrow \\
A_3^{(\alpha)} & \rightarrow & x_2, x_3, x_4, x_5 & \\
A_4^{(\alpha)} & \rightarrow & x_2, x_3, x_4, x_5 & \\
A_1^{(\alpha)}, A_2^{(\alpha)} & \rightarrow & x_1, x_3 & \leftarrow \\
A_1^{(\alpha)}, A_3^{(\alpha)} & \rightarrow & x_2, x_3 & \\
A_1^{(\alpha)}, A_4^{(\alpha)} & \rightarrow & x_2, x_3 & \\
A_2^{(\alpha)}, A_3^{(\alpha)} & \rightarrow & x_3, x_4, x_5 & \\
A_2^{(\alpha)}, A_4^{(\alpha)} & \rightarrow & x_3, x_4, x_5 & \\
A_3^{(\alpha)}, A_4^{(\alpha)} & \rightarrow & x_2, x_3, x_4, x_5 & \leftarrow \\
A_1^{(\alpha)}, A_2^{(\alpha)}, A_3^{(\alpha)} & \rightarrow & x_3 & \\
A_1^{(\alpha)}, A_2^{(\alpha)}, A_4^{(\alpha)} & \rightarrow & x_3 & \\
A_1^{(\alpha)}, A_3^{(\alpha)}, A_4^{(\alpha)} & \rightarrow & x_2, x_3 & \leftarrow \\
A_2^{(\alpha)}, A_3^{(\alpha)}, A_4^{(\alpha)} & \rightarrow & x_3, x_4, x_5 & \leftarrow \\
E_1 & \rightarrow & x_3 & \leftarrow
\end{array}$$

By choosing for all the elements of the power set E_2 the element of the power set of E_1 with the largest number of components, we get:

$$\begin{array}{lll}
E_1 & \rightarrow & x_3 \\
A_1^{(\alpha)}, A_3^{(\alpha)}, A_4^{(\alpha)} & \rightarrow & x_2, x_3 \\
A_2^{(\alpha)}, A_3^{(\alpha)}, A_4^{(\alpha)} & \rightarrow & x_3, x_4, x_5 \\
A_1^{(\alpha)}, A_2^{(\alpha)} & \rightarrow & x_1, x_3 \\
A_3^{(\alpha)}, A_4^{(\alpha)} & \rightarrow & x_2, x_3, x_4, x_5 \\
A_1^{(\alpha)} & \rightarrow & x_1, x_2, x_3 \\
A_2^{(\alpha)} & \rightarrow & x_1, x_3, x_4, x_5 \\
\varnothing & \rightarrow & E_2
\end{array}$$

As we can see, the result is the same as the one found with the new algorithm.

Sometimes, the topologies $T(E_1)$ and $T(E_2)$ are the same than the power set of both sets E_1 and E_2, as it happens in the following problem (Gil-Aluja and Gil-Lafuente, 2007):

	x_1	x_2	x_3	x_4
$A_1^{(\alpha)}$	1	1	1	
$A_2^{(\alpha)}$		1	1	
$A_3^{(\alpha)}$		1		1

We find the topology $T(E_2)$:

$$F^A = \{\{x_2, x_3\}, \{x_2, x_4\}, \{x_1, x_2, x_3\}\}$$
$$\bar{F}^A = \{\{x_1, x_4\}, \{x_1, x_3\}, \{x_4\}\}$$
$$F_1 \cap F_2 \cap F_3 = \{x_2\} \qquad F_1 \cap F_2 \cap \bar{F}_3 = \{x_3\}$$
$$F_1 \cap \bar{F}_2 \cap F_3 = \varnothing \qquad \bar{F}_1 \cap F_2 \cap F_3 = \varnothing$$
$$F_1 \cap \bar{F}_2 \cap \bar{F}_3 = \{x_1\} \qquad F_1 \cap \bar{F}_2 \cap F_3 = \bar{\varnothing}$$
$$\bar{F}_1 \cap \bar{F}_2 \cap F_3 = \{x_4\} \qquad \bar{F}_1 \cap \bar{F}_2 \cap \bar{F}_3 = \varnothing$$

$$T(E_2) = \{\{x_1\}, \{x_2\}, \{x_3\}, \{x_4\}, \{x_1, x_2\}, \{x_1, x_3\}, \{x_1, x_4\}, \{x_2, x_3\}, \{x_2, x_4\}, \{x_3, x_4\}, \{x_1, x_2, x_3\}, \{x_1, x_2, x_4\}, \{x_1, x_3, x_4\}, \{x_2, x_3, x_4\}, E_2\}$$

Now, we get the topology $T(E_1)$:

$$F^x = \left\{\{A_1^{(\alpha)}\}, \{A_1^{(\alpha)}, A_2^{(\alpha)}, A_3^{(\alpha)}\}, \{A_1^{(\alpha)}, A_2^{(\alpha)}\}, \{A_3^{(\alpha)}\}\right\}$$
$$\bar{F}^x = \left\{\{A_2^{(\alpha)}, A_3^{(\alpha)}\}, \varnothing, \{A_3^{(\alpha)}\}, \{A_1^{(\alpha)}, A_2^{(\alpha)}\}\right\}$$

$$F_1 \cap F_2 \cap F_3 \cap F_4 = \varnothing \qquad F_1 \cap F_2 \cap F_3 \cap \bar{F}_4 = A_1$$
$$F_1 \cap F_2 \cap \bar{F}_3 \cap F_4 = \varnothing \qquad F_1 \cap \bar{F}_2 \cap F_3 \cap F_4 = \varnothing$$
$$\bar{F}_1 \cap F_2 \cap F_3 \cap F_4 = \varnothing \qquad F_1 \cap F_2 \cap \bar{F}_3 \cap \bar{F}_4 = \varnothing$$
$$F_1 \cap \bar{F}_2 \cap F_3 \cap \bar{F}_4 = \varnothing \qquad F_1 \cap \bar{F}_2 \cap \bar{F}_3 \cap F_4 = \varnothing$$
$$\bar{F}_1 \cap F_2 \cap F_3 \cap \bar{F}_4 = A_2 \qquad \bar{F}_1 \cap F_2 \cap \bar{F}_3 \cap F_4 = A_3$$
$$\bar{F}_1 \cap \bar{F}_2 \cap F_3 \cap F_4 = \varnothing \qquad \bar{F}_1 \cap \bar{F}_2 \cap \bar{F}_3 \cap F_4 = \varnothing$$
$$\bar{F}_1 \cap F_2 \cap \bar{F}_3 \cap \bar{F}_4 = \varnothing \qquad \bar{F}_1 \cap \bar{F}_2 \cap F_3 \cap \bar{F}_4 = \varnothing$$
$$\bar{F}_1 \cap \bar{F}_2 \cap \bar{F}_3 \cap F_4 = \varnothing \qquad \bar{F}_1 \cap \bar{F}_2 \cap \bar{F}_3 \cap \bar{F}_4 = \varnothing$$

$$T(E_1) = \left\{\varnothing, \{A_1^{(\alpha)}\}, \{A_2^{(\alpha)}\}, \{A_3^{(\alpha)}\}, \{A_1^{(\alpha)}, A_2^{(\alpha)}\}, \{A_1^{(\alpha)}, A_3^{(\alpha)}\}, \{A_2^{(\alpha)}, A_3^{(\alpha)}\}, E_1\right\}$$

Lets see the relation between the two topologies, that in this case is the two power sets:

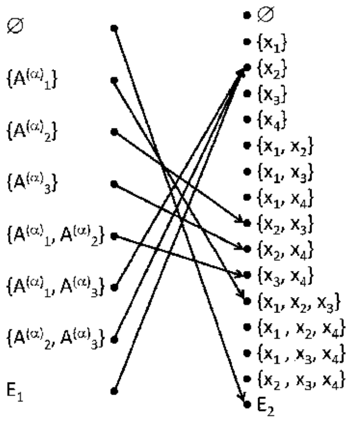

Fig. 7 Relation between the two topologies

The relations cannot be expressed, in this case, by using a Boolean lattice, but it is possible to represent them by using a Galois lattice (if we add to the relation $\{E_1, x_2\}$ the $\{E_1, \varnothing\}$), as follows:

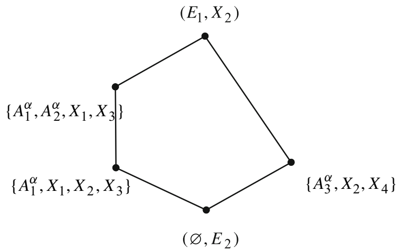

Fig. 8 Galois lattice

Note that this result is the same than the result that appears in the work (Gil-Aluja and Gil-Lafuente, 2007) that uses an assignment algorithm.

5 Algorithm for the Maximum Relations Between the Titles and the Attributes

Once it has been shown the relation between the two topologies $T(E_1)$ and $T(E_2)$, and the maximizing character of their construction, we will now develop an algorithm that permits to solve the problems of relation between the titles and the attributes. Then, this algorithm will be also able to solve the problem of homogeneous groupings between them with an optimizing character. For doing so, we suggest the following steps:

A Obtention of the topology $T(E_2)$

1. We describe the titles with fuzzy subsets of the referential of its attributes.
2. We establish a threshold subset of the same referential set of attributes, from where we can obtain the description of the titles by using fuzzy subsets.
3. We form the family F of sets of attributes, where each one of these elements comprises the attributes possessed by each title. This family will comprise as much elements as titles it has.
4. For each element of the family F we get its complementary. We will also find as much complimentaries as titles, that will form the family $\bar{F}$.
5. We develop all the possible intersections between the elements of the family F and the elements of the family $\bar{F}$. We select the non empty intersections.
6. These non empty intersections, (each one of them contains one or more attributes) are connected in all the possible ways and we add the empty set. Then, we get a topology $T(E_2)$.

B Obtention of the topology $T(E_1)$

7. The attributes are described by using fuzzy subsets of the referential set of the titles.
8. Using the same threshold subset, we get the description of each attribute by using Boolean subsets.
9. We form the family of subsets of tittles, the family of complimentaries and we develop all the possible intersections between them in a similar way as described in Steps 3, 4 and 5.
10. We select the non empty intersections (each of them contain at least one title), we develop all the possible unions and we add the empty set, obtaining the topology $T(E_1)$.

C Establishment of relations between two topologies

11. We go back to the beginning to establish the Boolean relations between titles and attributes, by using a graph in order to obtain a better visualization.
12. We select between the two topologies the one with a lowest number of elements in the topology $T(E_1)$. Each element of this topology will be related with the elements of the power set of the other set, in our case E_2 (the attributes), that are possessed by all the titles. We can prove that each of the groups of attributes found are part of the topology $T(E_2)$.
13. In the case that one element of the topology $T(E_1)$ is related with more than one element of the topology $T(E_2)$, then, we select the element with a largest number of attributes.
14. The relation obtained constitutes an optimal that, moreover, permits the grouping of titles with the highest number of shared attributes. It also permits the grouping of attributes that are possessed together by the highest number of titles.

6 Conclusions

The operators that work in the financial markets usually try to form groups of titles that permit to obtain some qualities such as rentability, stability, risk limitation, etc., in order to be prepared against an unexpected volatility.

It is obvious that there exist a wide range of methods for selecting the titles of a portfolio. In this paper, we have considered a scenario where we form in a first stage, subsets of titles, where each one of them is constituted by elements that possess certain homogeneity according to some attributes previously established. Note that the objective is to extract one or several titles of each subset in order to get the desired properties.

The theoretical solution of an approach as the one described here, has generalized other previous algorithms that were not so complete as this one such as the Pichat algorithm or the Kaufmann and Gil-Aluja algorithm.

We believe that the presented algorithm solves the problems of the Pichat and the Kaufmann and Gil-Aluja algorithms. Note that in some exceptional situations the algorithm presented here can become the Kaufmann and Gil-Aluja algorithm. This happens when the two topologies used in the relation are equal to the power sets of the sets of titles and its attributes.

In the beginning of this work we have shown two methods for fuzzifying the topologies. The approach developed in this paper is based on one of these methods. Note that in future research, we will consider the possibility of using the other method in the problem.

Finally, we want to mention the usefulness in the operations of our algorithm that has been tested with satisfactory results twice with the examples shown in the paper.

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Elman Nets for Credit Risk Assessment

Giacomo di Tollo and Marianna Lyra

Abstract. Nowadays the correct assessment of credit risk is of particular importance for all financial institutions to ensure their stability. Thus, Basel II Accord on Banking Supervision legislates the framework for credit risk assessment. Linear scoring models have been developed for this assessment, which are functions of systematic and idiosyncratic factors. Among statistical techniques that have been applied for factor and weight selection, Neural Networks (NN) have shown superior performance as they are able to learn non linear relationships among factors and they are more efficient in the presence of noisy or incorrect data. In particular, Recurrent Neural Networks (RNN) are useful when we have at hand historical series as they are able to grasp the data's temporal dynamics. In this work, we describe an application of RNN to credit risk assessment. RNN (specifically, Elman networks) are compared with two former Neural Network systems, one with a standard feed-forward network, while the other with a special purpose architecture. The application is tested on real-world data, related to Italian small firms. We show that NN can be very successful in credit risk assessment if used jointly with a careful data analysis, pre-processing and training.

1 Introduction

In banks' and other financial institutions' daily work, credit risk is the likelihood that a debtor will not fulfil his/her obligation, and thus credit risk management is essential for all banking institutions. The assessment of insolvency risk plays an

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important role since it helps banks decide whether to grant the requested loan or not. Several methods and techniques can be used to tackle this decision, the most common being linear scoring models. Among them, nowadays, Neural Networks (NN) are gaining more and more success and importance mainly because of their capability to learn non-linear relationships amongst variables; moreover, they show good performances when data are noisy or incorrect.

In this work we apply neural nets[1] to the insolvency prediction, in order to classify firms in two distinct sets: *default* firms (the ones that are not expected to repay the loan at the end of the period) and *in bonis* firms (that are supposed to fulfil their obligation). We use real-world data, after operating a series of pre-processing operations in order to make it suitable for our purposes. The neural nets parameters are set using an optimisation procedure analogous to the gradient descent (Angelini et al., 2008).

In our experiments, we use Recurrent Neural Networks (RNN). The basic feature of this topology is that it allows connections between units forming directed cycles. This allows the net to exhibit dynamic temporal behaviour, and it has shown good results when applied to difficult prediction tasks, such as the one at hand.

The paper is organized as follows. Section 2 reports on banks' credit risk assessment according to Basel II Capital Accord. The legal framework of Basel II is summarized. Section 3 introduces NN learning techniques and their basic properties. Previous academic results on the application of NN in economic/financial areas are presented as well as the major contribution of the paper. Section 4 describes the data and addresses the data pre-processing techniques. Section 5 reports on the experimental procedures, while Section 6 discusses the major results. Finally, Section 7 concludes.

2 Credit Risk

Credit risk refers to the likelihood that a borrower will not fulfil his/her lending obligations. Banking institutions are interested in forecasting the insolvency risk (default likelihood) of a potential borrower so as to better allocate financial resources in a portfolio and to better assign the appropriate lending interest rates to borrowers. On one hand, if the riskiness of a borrower is overstated the interest rate offered by the bank might not be as attractive as the one offered by a competitive bank. On the other hand, if the riskiness of a borrower is understated the bank will soon come across unexpected losses in excess of its provisions.

The development of a credit model is vital for forecasting borrowers' insolvency risk. The aforementioned model is a function of observable and unobservable weighted risk factors. Its derivation is subject to a diverse range of risk factors and their weights. Hamerle et al. (2003) report a generalized factor model for credit risk

[1] Neural Networks (NN).

composed by firm specific financial factors, economic factors and their lags, systematic and idiosyncratic factors. For credit risk modeling and weight selection, statistical techniques vary among linear discriminant analysis (Altman, 1968; Altman et al., 1994), conventional stepwise analysis (Butera and Faff, 2006), logit analysis, NN (Angelini et al., 2008) and genetic algorithms (Back et al., 1996).

Technical instruments, like NN, have been widely applied for credit risk assessment. They rather serve as default advisors (default–non default) rather than tools for the exact estimation of default likelihood. Furthermore, they are recognised as appropriate tools linked to the Basel II's legislation framework for banks credit risk assessment.

2.1 Basel Committee on Banking Supervision

The Basel II Capital Accord on Banking Supervision legislates the framework under which the stability of financial institutions is ensured (Basel Committee on Banking Supervision, 2006). Three supplementary parts build up the pyramid of Basel II's framework, minimum capital requirements, authorities' supervision of capital adequacy and bank's disclosures for market supervision. On the base of the pyramid there is the adequate calculation of bank's minimum required capital to account for credit, operational and market risk. We emphasise on credit risk.

There are two alternative approaches that a bank can apply to determine the adequate capital level for credit risk, namely standard approach and internal rating-based approach (IRB). A general requirement for both methods is that the minimum solvency ratio should be greater than 8%. The solvency ratio is the capital requirement for assets' credit risk divided by 'risk-weighted assets'.

$$\text{Solvency ratio} = \frac{\text{capital requirement}}{\text{risk-weighted assets}} \geq 8\% \,. \tag{1}$$

Under the standard approach, in order to compute the capital requirement, credit risk ratings or risk weights for asset values are determined by an external credit rating agency. However, under the IRB approach capital requirement for credit risk is estimated internally by the bank. The IRB approach, Basel Committee on Banking Supervision (2006), § 258, classifies bank's assets to sovereign, bank, corporate, retail and equity. For each asset class a different capital is required.

Banks are required to detain a minimum capital level to cover, with confidence 99.9%, unexpected losses from defaulted borrowers. Unexpected losses (UL) might occur from economic depression. While banks can cover expected losses through provisions, losses from unexpected economic depression are covered with capital requirements.

Expected loss amount (EL) equals the product of the exposure at default (EAD), the loss given default (LGD) and the likelihood that a borrower's default will occur in the subsequent year (PD). Basel II requires banks to hold a regulatory capital to cover total realised losses from extraordinary events (TL) that might exceed provisions (EL).

That is,

$$\mathrm{UL} = 1.06 \cdot (\mathrm{TL} - \mathrm{EL})\,, \tag{2}$$

$$\mathrm{TL} = \Phi\left(\frac{\Phi^{-1}(\mathrm{PD}) - \sqrt{\mathrm{R}} \cdot \Phi^{-1}(0.001)}{\sqrt{1-\mathrm{R}}}\right) \cdot \mathrm{EAD} \cdot \mathrm{LGD}\,. \tag{3}$$

Asset correlation (R) determines the extent to which the assets of a portfolio are exposed to systematic risk factors such as macroeconomic risk factors. For sovereign, corporate, and bank asset classes the asset correlation is given by

$$\mathrm{R} = 0.12 \cdot \left(\frac{-\exp(-50 \cdot \mathrm{PD})}{1-\exp(-50)}\right) + 0.24 \cdot \left(1 - \frac{1-\exp(-50 \cdot \mathrm{PD})}{1-\exp(-50)}\right) - 0.04 \cdot \left(1 - \frac{S-5}{45}\right)\,. \tag{4}$$

Equation 4 introduces a firm-size adjustment to asset correlation (R) for exposures of small and medium size (SME) firms (exposures between EUR 5 million and EUR 50 million). The total amount of yearly sales (S) measures firm size and ranges between EUR 5 million and EUR 50 million. For large firms (total sales higher than EUR 50 million) the last part of the equation is eliminated. In our experiments we neglect the low correlation between SME (see Section 4). Hence, exposures below EUR 1 million are treated as retail exposures, as stated in Paragraph 232 of the Basel II.

Banks can either estimate only the PD, ‘Foundation IRB’, or estimate internally all the above risk components, ‘Advanced IRB’. In Eq. 2 the multiplier 1.06 controls for the possible underestimation of UL because of the advanced IRB approach. A qualifying IRB approach requires the design of a risk rating system for credit risk assignment, quantification and validation, that fulfils specific minimum requirements. On one hand, banks’ borrowers should be assigned to homogenous grades according to their risk exposures (PD). Then, for retail, all borrowers in each grade are assigned the same PD, LGD and EAD. On the other hand, certain borrower and transaction factors, i.e., collateral, seniority and type, must be considered.

In order for a bank to adequately calculate its capital requirements a reliable credit risk quantification technique should be applied. The design of a risk rating system for credit risk quantification can be done with one (or a combination) of the following techniques; historical default experience from identical risk borrowers, alignment to an agency’s credit rating scales and statistical credit scoring model. Of particular importance is the last technique where the model is borrower oriented: its derivation is subject to a diverse range of risk factors and their weights. For credit scoring model and weight selection, statistical techniques or machine-driven techniques, like NN, together with human judgment and IT systems shall be applied. NN serve as default advisors (default–non default) rather than tools for the exact estimation of default likelihood.

3 Neural Networks

NN are learning strategies designed and implemented in order to grasp relationships between input and output variables, assuming these relationships to be non-linear. They are considered to be black-boxes since it is not possible to extract symbolical knowledge neither from their current state, nor from their dynamic behaviour over time: the network solves the problem without explaining which procedure has been used. In this section, NN are introduced to define the background needed to discuss the proposed model and the experimental phase.

3.1 An Introduction to Neural Networks

The Artificial Neural Networks' (NN) behaviour is inspired by Biological Neural Networks (BNN). They are composed of elements working in parallel (neurons) which, taken as single units, are only able to perform just simple tasks: they process information coming from other neurons via an activation function and they communicate the outcome of this function to other connected elements. The integration of these simple tasks via distributed computing makes them suitable to execute demanding tasks.

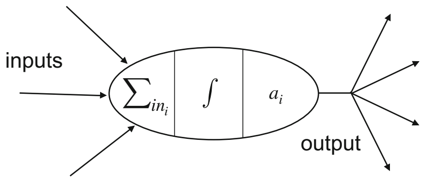

Fig. 1 Artificial neuron basic structure

Without entering into BNN details, we can say that NN are composed of elemental units (neurons, see Fig. 1) and of weighted and oriented links between them (synapses). A value (a_i) is assigned to each neuron which is later transferred to connected elements. This value is determined w.r.t. the input values received from other neurons ($\sum_{in_i}$) via synapses, the activation and the output functions. Each synapsis is also associated a value, being either a positive or negative one, which determines the magnitude of the transmission that is subject to learning over time.

Depending to their functions, artificial neurons can be classified as

- Inputs neurons, meaning neurons whose activation values represent the instance inputs.
- Output neurons, meaning neurons whose activation values represent the instance output (solution).
- Hidden neurons, that are not visible outside the network, the task of which is just a computational one.

The network's behaviour is determined by

- Activation Function, which determines the neuron's output value starting from connected neurons' activation values.
- Synapses, which determine the activation quantity to be transferred to connected neurons.
- Topology, which is the way neurons are connected to each other.
- Temporal Dynamics, which determine when to update neurons' activation values and the criterion to use to this extent (i.e., whether to update all neurons at a time or to update just a subset: in the latter case, a criterion must be introduced to formalise the choice of the subset).

These factors can either be determined by the user or learnt by the network itself. Generally, a hybrid approach is applied, as some features are determined *a priori* by the user and others by the learning procedure. Historically, synapsis' weights have always been subject to learning procedures leading to the development of learning algorithms which became more and more popular ever since, i.e., Backpropagation. Lately, learning paradigms have also been applied to other parameters, also due to new Artificial Intelligence techniques allowing, for example, to determine the best net architecture. Furthermore, it should be stressed out that generally learning algorithms are developed for a specific net architecture: this means that generally the architecture choice greatly influences the algorithm to be used.

Basic neurons may be organised in several architectures to build a Neural Network. A first architecture proposes neurons completely connected to each other, so as to determine a completely connected structure, as in Fig. 2a. Another architecture proposes neurons to be grouped in several layers, conceived as disjoints and ordered subsets, depending on their functions: we have an *input* layer, one or more *hidden* layers and an *output* layer. Each layer is connected to neurons belonging to adjacent layers.

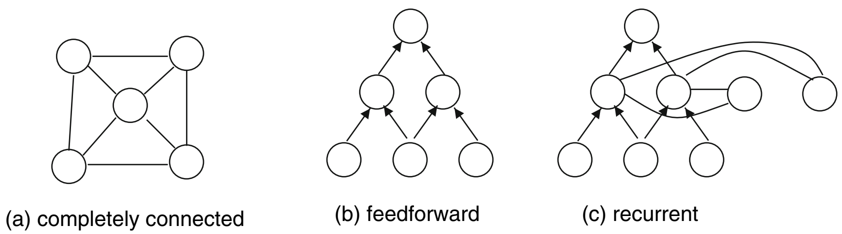

Fig. 2a,b,c Examples of network topologies

In the most common model, every neuron of one layer is connected to all neurons of an adjacent layer, while there are no connections between neurons of the same layer. Input neurons have no ingoing connections, whereas their activations are the input data of the problem. A function transfers these activations (without any computation) to neurons belonging to the first hidden layer. Neurons compute their activation function and they transfer it either to the next hidden layer, or to the output layer. The information flow is *feed-forward*, as neurons receive information from the previous layer and transfer their activation only to the next layer. Because of this feature, nets of this kind are referred to as *feed-forward* (Fig. 2b).

RNN represent a variant of the layered structure introduced previously in which a loop can be identified in the information flow. The most common are Jordan and Elmans nets. In Jordan nets, there exist *state units* which act as input units (with self-connections), receiving inputs directly from the output units; in Elman nets instead, in top of a feed-forward network, we add a further layer composed of *context units*, which receive input from the hidden layer and transfer their output to the very same layer (Fig. 2c). Also these units may have self-connections.

3.2 Learning Algorithms

To apply NN to the problem, a first phase consists in tuning its synapsis' weights. This task is accomplished by a learning algorithm, which trains the network by iteratively modifying the synapsis' weights until a termination condition is met. In most applications, the learning algorithm stops as soon as the discrepancy (*error*) between the desired output and the output produced by the network falls below a predefined threshold. There are three typologies of learning mechanisms for NN:

- Supervised learning.
- Unsupervised learning.
- Reinforced learning.

Supervised learning is characterised by a *training set* of correct examples used to train the network. The training set is composed of pairs of inputs and corresponding desired outputs. The error produced by the network is used to change the weights. Generally, the error is evaluated on a latter set, the *test set*, and the algorithm stops when the *error* produced on this latter set falls below a given threshold. This kind of learning is applied in cases where the network has to generalise the given examples. A typical application is the classification of inputs: A given input has to be inserted in one of the defined categories.

In *unsupervised learning* algorithms, the network is only provided with a set of inputs and no desired output is given. The algorithm guides the network to self-organise and to adapt its weights. This kind of learning is used for tasks such as data mining and clustering, where some regularities in a large amount of data have to be found.

Finally, *reinforced learning* trains the network by introducing prizes and penalties as a function of the network response. Prizes and penalties are then used to modify the weights. Reinforced learning algorithms are applied, for instance, to train adaptive systems which perform a task composed of a sequence of actions.

The final outcome is the result of this sequence, therefore the contribution of each action has to be evaluated in the context of the action chain produced.[2]

Diverse algorithms to train NN have been presented in the literature. There are algorithms specifically designed for a particular kind of NN, such as the *backpropagation algorithm* (Rumelhart et al., 1986), which has become the most widely used algorithm, or general purpose algorithms, such as *genetic algorithms* (Mitchell, 1996) and *simulated annealing* (Kirkpatrick et al., 1983).

It is of particular importance to remark the main advantages and limitations of NN systems. The main advantages are on the one hand their learning capabilities and on the other hand the fact that the derived model does not make any assumptions about the relationship amongst input variables. Conversely, a theoretical limitation of NN is that they are *black-box* systems and the extraction of symbolic knowledge is awkward. Moreover, the design and the optimisation of NN methodologies are mainly empirical. Thus, the experience and sensibility of the designer have a strong contribution to the final success of the application. Nevertheless, this work contributes to the literature by showing that there exist some useful general design and parameters optimisation guidelines.

3.3 Related Works

After their introduction in the 1940s (McCulloch and Pitts, 1943), NN have been applied in several fields: weather forecasting, pattern recognition, robotics, data smoothing, etc. Their application to the economical-financial area is more recent, as it was pursued starting from the 1980s. It is possible to classify problems arising in this area as follows:

1. Classification.
2. Temporal series forecasting.
3. Function approximation.

Credit risk assessment belongs to the second class,[3] and NN have been widely applied to this problem (Hamid and Iqbal, 2004; Rong-Zhou et al., 2002; Pang et al., 2002; Wu and Wang, 2000; Piramuthu, 1999) in order to be compared with standard econometric tools (Altman, 1968), generally showing better performances. To measure performance, the percentage of wrongly classified firms, meaning the overall errors, over the test set is calculated.[4] The overall error is not a good estimator of the net's generalization capabilities. In fact, as stated in Section 1, errors can be partitioned in two classes:

[2] In these cases it is difficult, if not impossible, to produce an effective training set, hence the need of the use of prizes and penalties, instead of the definition of an explicit error as in the case of supervised learning.

[3] The main example of the first class is asset prices prediction, whilst the third class represents a residual class for problems neither in the first class nor in the second, i.g., pricing exotic options.

[4] Results are reported in Atiya, 2001.

1. *misbo*, indicating that the net wrongly classifies firms as in *default* when actually they are *in bonis*.
2. *misdef* indicating that the net wrongly classifies firms as *in bonis* when actually they are in *default*.

The second class of errors is the most important for banks as its presence means granting the loan to an unsafe firm. For this reason, it is important to distinguish between these two classes when measuring the NN performance. Yet, there is only limited work Zone in this area. Table 1 illustrates previous results for this binary classification reporting, for each work, the percentage of wrongly classified *in bonis* and *default* firms (*misbo* and *misdef*) as a percentage over the test set.

Table 1 Statum of the art

	Test set misbo %	Test set misdef %
Wu and Wang, 2000	0	57.01
Jagielska and Jaworski, 1996	29.86	32.38
Pang et al., 2002	2.33	0
Angelini et al., 2008	13.3	0
Angelini et al., 2008	0	12.5

It is prevalent that NN exhibit difficulties in learning the concept of *default*, as shown by the high percentage of *misdef*. This inefficiency has been the key assumption in the previous work of Angelini et al. (2008), which used two net topologies obtaining better (or comparable) results than the competitors' ones reported in Table 1 (for this reason this work is cited in two entries in Table 1, and it is worthwhile to notice that the two nets are complimentary, as the first correctly classifies *default* firms, whilst the second correctly classifies *bonis* firms).

Specifically, results reported in Table 1 (with the notable exception of Angelini et al., 2008) must be taken *cum grano salis*, as they are obtained just by using a single partition of data between training test and test set. In our work instead, we want to take into account the general robustness of the devised NN, so we introduce several partitions of data at hand and we devise statistical measures in order to directly assess NNs' behavior.[5] The current work aims at introducing RNN to obtain even better results than that of the outstanding literature.

It is worthwhile noticing that there are several studies that compare NN with more accredited linear models, but the final verdict of this comparison is far from being univoque: NN outperform linear methods such as Logistic and Probit in (Fan and Palaniswami, 2000; Galindo and Tamayo, 2000; Salchenberger et al., 1992; Odon and Sharda, 1990). Some studies claim the superiority of linear methods (Yang et al., 1999; Altman et al., 1994), while others still lead to un-unique results (Boritz and Kennedy, 1995; Coats and Fant, 1993).

[5] The same approach has been followed by Angelini et al. (2008).

What is interesting is that assessing the superiority of a method over another one is usually based on several non-homogeneous criteria: in such a case it is difficult, if not meaningless, to try to assess the general superiority of a technique over another one.

4 Data Set

For our experiments we used data of 76 small firms, clients of an Italian bank. The sample is rather diversified and distributed across industries. Small firms are an important feature of the Italian economy. Therefore, we apply the NN method for banks' lending decisions to the small size firms.

Generally, it is difficult to collect data from small firms, and even when available it is often noisy. For this reason, banks have turned to NN for assistance in making lending decisions more accurate, faster and cheaper. For each firm we have annual data across three years (2001–2003)[6], so that the sample period covers three years. For each firm there are 15 indices: 8 of them are financial ratios drawn from all ratio classes (Liquidity ratios, Asset Turnover ratios, Leverage ratios, Profitability ratios)

- Cash flow/Total debt.
- Net working capital/Total assets.
- Turnover/Inventory.
- Trade accounts receivables/Turnover.
- Current Liability/Turnover.
- Equity/Total assets.
- Financial costs/Total debts.
- Value added/Total assets.

The remaining 7 are credit-positions-history ratios calculated by analysing the credit positions with the supplying bank (*'Andamentale'* ratios):

- Utilized credit line/Accorded credit line,
- Unsolved effects (quantity (q))/Under usual reserve effects (q),
- Unsolved effects (value (v))/Under usual reserve effects (v),

and with the overall Italian Banking System (*'Centrale dei Rischi'* ratios):

- Transpassing short (s) term/Accorded credit line s term,
- Transpassing medium-long (m-l) term/Accorded credit line m-l term,
- Utilised credit line s term/Accorded credit line s term,
- Utilised credit line m-l term/Accorded credit line m-l term.

Up to our knowledge the ratios of the latter group (credit-positions-history ratios) have not been used in the previous literature (see Atiya, 2001). Statistical features of the data set at hand can be found in Table 2.

[6] This period may happen to be a down turn (or up turn) in a cycle. External factors such as business effects and the macroeconomic environment can be neglected for the low correlation between small and medium exposures (SME) and the economic cycle.

Table 2 Ratios' statistical features before pre-processing operations

Index	Min	Max	Avg	Correct entries
Cash flow/Total debt	–1.07	0.52	0.04	315
Turnover/Inventory	0	1877.00	77.95	233
Current Liability/Turnover	0	1277.50	11.35	312
Equity/Total assets	–10.69	1.60	0.03	305
Financial costs/Total debts	0	0.72	0.05	315
Net working capital/Total assets	–78.75	1.01	–0.82	305
Trade accounts receivables/Turnover	0	13.84	0.51	312
Value added/Total assets	–0.22	1.13	0.24	305
Utilized credit line/Accorded credit line	0	14.54	0.81	219
Unsolved effects (q)/Under usual reserve effects (q)	0	4.48	0.48	39
Unsolved effects (v)/Under usual reserve effects (v)	0	5.15	0.52	39
Transpassing s/Accorded credit line s	–2.74	18.95	0.21	156
Transpassing m-l/Accorded credit line m-l	0	3.99	0.11	207
Utilized credit line s/Accorded credit line s	–0.06	19.95	0.80	157
Utilized credit line m-l/Accorded credit line m-l	0	4.99	0.85	208

The sample firms are split in two groups: The *in bonis* group (firms repaying their loan obligations at the end of the analysing period) and the *default* group (conversely, firms not repaying their loans at the end of the period).

4.1 Data Pre-Processing

A crucial phase in the deployment of a NN-based application is data pre-processing, which is often performed without a systematic methodology (despite the several issues that have been already discussed by Bishop, 2005). Therefore, in our work we apply a simple, yet complete, data pre-processing procedure, that is explained in more detail in what follows. This aspect is proven to be an important parameter for NN's performance.

4.1.1 Missing and Wrong Values

Some values were missing or were incorrect in the database. The most common way to overcome this situation is to eliminate from the dataset all entries of the corresponding firm, but this would lead to a significant loss of information. In order to preserve as much information as we could, we replaced the missing values with 'meaningful' ones: If a value was missing or was clearly wrong we decided to substitute the empty space with the arithmetical mean of the whole index. It is the mean of all firms' values over the collecting period corresponding to the particular index. Otherwise, if the value was missing because of a computational error (i.e. variables are mathematical ratios, and in our sample some ratios can derive by a division by zero) we decided to replace the missing value with the upper limit of the normalised interval (see Section 4.1.3).

This choice can be understood by what follows: as the computational error occurs when dividing the ratio by zero, we can imagine the result of this ratio being ∞ (infinitive). If we normalise the corresponding index, this occurrence will be replaced by the maximum value in the range (we will see, this value being 1). Objections about this approach can raise if both the numerator and the denominator in the ratio were equal to 0, but no firms present such kind of property and indeed, due to the nature of the indices, the likelihood of both the numerator and the denominator being 0 is negligible. For the sake of completeness, we point out that the eight *Balance Sheet* ratios are the most 'clean': They present a tiny portion of wrong values and no missing data. *Credit-positions-history ratios* are conversely more noisy, as they present more missing and wrong data. Interestingly, wrong observations in *Balance ratios* are due to both inconsitency and computational errors, whilst *Credit-history* ratios suffer only from computational errors.

4.1.2 Erasing Useless Indices

Replacing missing and wrong values can lead to learning inconsistencies when the observed attribute contains many missing or wrong values: in this case the net would not be able to generalise starting from data at hand, as they are few and far between.

Furthermore, as we will see, the sample is split into *training* and *test* set (see Section 3.2) using some percentage ratios (we will use [70% 30%]). In the worst case scenario, if a specific index has more than 30% of missing and wrong values over the total number of observations, they are all included in the test set. This will result in a tremendous loss in net's generalisation skill. For this reason we decided to erase from our analysis indices containing more than 30% of missing and wrong values which are reported in what follows. These indices belong to 'Andamentale' and 'Centrale dei Rischi' ratios:

1. Unsolved effects (quantity)/Under usual reserve effects (quantity).
2. Unsolved effects (value)/Under usual reserve effects (value).
3. Transpassing s term/Accorded credit line s term.
4. Utilized credit line s term/Accorded credit line s term.

4.1.3 Data Normalisation

Normalisation is required to feed the net with homogeneously ranged data for each input node. We used the interval [0, 1] for each node. Please note that by using standard normalisation techniques useful information might be lost due to the deviation from the index mean (Table 2). Further, the average value of some indices is very close to their minimum or maximum value, i.e, *Turnover/Inventory, Current Liability/Turnover, Utilized credit line s term/Accorded credit line s term, Utilized credit* *line m-l term/Accorded credit line m-l term* or *Net working capital/Total assets*, respectively. However, these ratios cannot be said to be incorrect and have important financial meaning, so we decided to use a logarithmic formula to normalise them. This formula is user-defined, thus it is very flexible:

$$\overline{x} = \log_u (x + 1)\,. \tag{5}$$

In the formula, $\overline{x}$ represents the normalised value, x is the value before normalisation and u is chosen by adding the value 1 to the maximum value belonging to the index. For the formula to be valid, $\overline{x}$ must be always ≥ 1, so we add 1 to the index value, assuming its lower bound being 0 without loss of generality.

4.1.4 Correlation Analysis

Lastly, a Pearson correlation analysis was performed to identify the most strongly correlated variables and remove them from later experiments. The analysis resulted in no strong correlation between pairs of variables, so we feed the NN with the 11 indices examined after erasing useless indices (see Section 4.1.2). Statistical features of the dataset after the pre-processing phase can be found in Table 3.

Table 3 Ratios' statistical features after pre-processing operations

Index	Min	Max	Avg	Total entries
Cash flow/Total debt	0	0.76	0.52	318
Turnover/Inventory	0	1	0.58	318
Current Liability/Turnover	0	1	0.13	318
Equity/Total assets	0	1	0.27	318
Financial costs/Total debts	0	0.72	0.05	318
Net working capital/Total assets	0	1	0.42	318
Trade accounts receivables/Turnover	0	1	0.11	318
Value added/Total assets	0.14	1	0.40	318
Utilized credit line/Accorded credit line	0	1	0.44	318
Transpassing ml/Accorded credit line ml	0	1	0.14	318
Utilized credit line ml/Accorded credit line ml	0	1	0.39	318

5 Experimental Analysis

In the experiments we used two topologies of Elman's RNN: The first is obtained just by adding a context layer to a standard three layered feed-forward network. For this reason it is called *Standard Elman* (see Fig. 3).

The second topology of RNN is called *Ad-hoc Elman* and is obtained by adding a context layer to a four layered feed-forward in which input neurons are first grouped by three, while the next layer connections are feed-forward. Each input group is connected to one neuron of the following layer. For the analysis two versions of this network are used: in the first the context layer is connected with the first hidden layer (see Fig. 4), whereas in the second the context layer is connected with the second hidden layer (see Fig. 5).

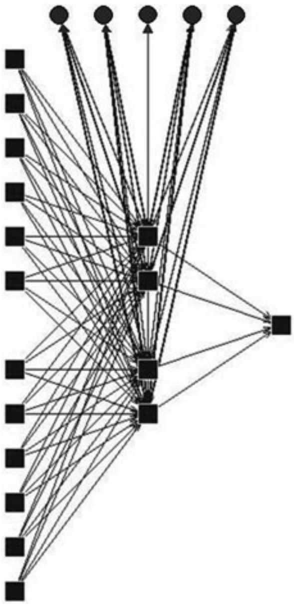

Fig. 3 Standard Elman network. Context units are shown in *circles*

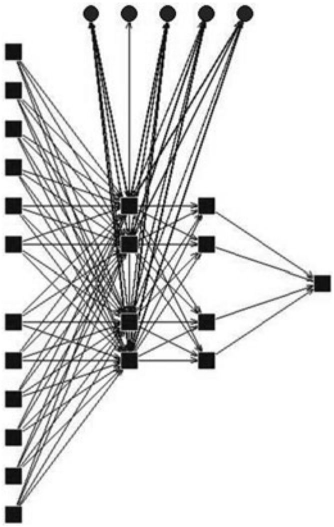

Fig. 4 First Ad-hoc Elman network. Context units are shown in *circles*

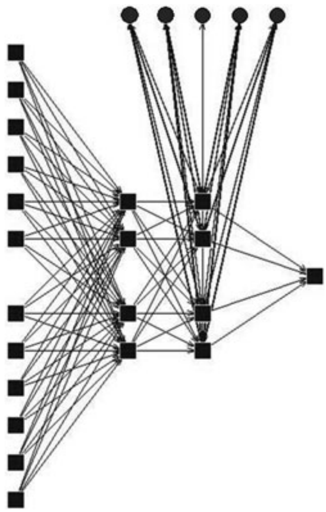

Fig. 5 Second Ad-hoc Elman network. Context units are shown in *circles*

Please note that these networks are straightforward extensions of the two basic architectures introduced in Angelini et al. (2008) obtained by adding a context layer to define a recurrent architecture. Both networks are trained by using a backpropagation algorithm, a supervised algorithm that performs an optimisation of the network's weights by minimising the error between the desired and the actual output. This learning schema is characterized by a *training set* (the set of correct examples used to train the network, composed of pairs of inputs and corresponding desired outputs) and a *test set* (used to test and evaluate the performance of the network). The training set is composed of 70% of the overall sample, and the remaining 30% composes the test set (this partition is widely used, see Atiya, 2001).

The inputs of the network are the eleven normalised attributes. In the *Standard Elman* network, they are simply given as an ordered array, while in the *Ad-hoc Elman* network they are first grouped in triads, where each group corresponds to the values of an attribute over 3 years.

The output y of the network is a real value in the range $[0, 1]$ interpreted as follows:

- If $y < 0.5$ then the firm is classified as *bonis*;
- otherwise, the input is classified as *default*.

To ensure the consistency of the results we run 30 experiments for different analogies between *training* and *test* sets. Each network has been trained and tested with every possible combination of training and test set. Results for each partition of data have been pooled together to draw statistical inference.

Tables 4 and 5 report results based on the previous work of Angelini et al. (2008) who used a standard feed-forward net and an ad-hoc network with input neurons grouped in triads. The tables report the misclassification error, in percentage points,

Table 4 Feedforward networks: statistics over 30 different analogies between *training* and *test* sets set

Hidden neurons %	Mean Misbo %	Std Misbo %	Mean Misdef %	Std Misdef %	Mean Overall %	Std Overall %
25	10.53	11.59	14.00	11.19	11.74	8.96
26	12.13	11.87	13.00	11.83	12.43	9.50
27	12.80	12.83	14.00	11.47	13.22	9.89
28	10.27	9.84	14.00	9.98	11.57	7.93
29	10.93	10.41	12.75	9.61	11.57	8.58
30	12.13	11.87	13.25	11.68	12.52	8.94
31	13.20	10.89	14.75	11.76	13.74	8.84
32	11.60	12.03	12.25	11.14	11.83	9.58
33	13.60	12.34	16.00	11.58	14.43	9.43

Table 5 Ad hoc networks: statistics over 30 different analogies between *training* and *test* sets set

Hidden neurons	Mean Misbo	Std Misbo	Mean Misdef	Std Misdef	Mean Overall	Std Overall
11 + 11 (2 layers)	6.13	7.59	9.50	10.58	7.30	6.59

Table 6 Statistics for Ad-Hoc Elman Nets

Context neurons	Mean Misbo	Std Misbo	Max Misbo	Mean Misdef	Std Misdef	Max Misdef	Mean Overall	Std Overall	Max Overall
1:1	6.27	7.43	33.33	8.61	8.05	37.50	7.48	6.09	30.44
1:2	5.73	6.74	33.33	7.77	7.75	30.44	6.78	5.90	30.40
2:1	5.87	7.21	33.33	8.65	8.39	50.00	7.30	6.05	26.09
2:2	5.47	6.56	33.33	6.84	8.13	37.50	6.17	6.09	30.44
3:1	4.93	6.14	26.67	7.84	8.35	37.50	6.44	5.43	21.74
3:2	5.73	7.38	40.00	7.60	7.88	30.44	6.70	6.16	30.44
4:1	5.73	8.19	33.33	7.94	8.41	37.50	6.87	7.14	30.44
4:2	6.40	8.08	40.00	8.15	8.36	37.50	7.30	6.11	26.09
5:1	6.00	8.22	40.00	8.70	9.09	50.00	7.39	6.99	30.44
5:2	6.13	7.72	40.00	9.08	7.97	37.50	7.65	5.73	26.09
6:1	6.13	8.39	40.00	9.58	11.36	75.00	7.91	8.27	43.48
6:2	6.80	8.02	33.33	9.13	9.36	37.50	8.00	7.42	34.78
7:1	4.93	6.43	33.33	7.84	7.86	37.50	6.44	5.71	30.44
7:2	5.87	5.97	20.00	7.47	7.31	37.50	6.70	5.14	21.74
8:1	5.33	6.87	33.33	8.31	6.18	25.00	6.87	4.49	21.74
8:2	6.00	7.41	33.33	9.20	8.03	37.50	7.65	5.46	21.74
9:1	5.87	6.55	33.33	8.15	6.95	37.50	7.04	5.11	26.09
9:2	5.33	6.46	33.33	7.30	7.57	37.50	6.35	5.21	21.74
10:1	5.47	7.82	40.00	8.19	8.84	37.50	6.87	6.98	34.78
10:2	6.80	7.44	33.33	8.62	8.04	26.09	7.74	6.53	26.09
11:1	6.80	7.56	40.00	8.11	8.60	50.00	7.48	6.21	30.44
11:2	6.13	7.11	33.33	8.07	7.54	26.09	7.13	5.47	26.09

over the test set. The mean and the standard deviation of *misbonis*, *misdefault* and overall sample classification are reported.

The results for the feed-forward network are very robust with respect to the number of hidden neurons. The average total errors are between 11% and 14%. The ad hoc network has a total average error of about 7%.

Tables 6 and 7 show results for the two new RNN approaches. It is obvious that these approaches perform better than the *standard* feed-forward ones, in both *misbonis* and *misdefault* terms. These nets are not much robust with regard to neurons in the context layer[7] but the ad-hoc Elman network performs better than its *ancestor*

Table 7 Statistics for Standard Elman Nets

Context neurons	Mean Misbo	Std Misbo	Max Misbo	Mean Misdef	Std Misdef	Max Misdef	Mean Overall	Std Overall	Max Overall
1	9.71	10.21	46.67	12.23	10.04	62.50	11.01	8.24	43.48
2	9.08	10.06	46.67	11.80	9.96	62.50	10.48	8.26	43.48
3	8.72	9.31	40.00	12.08	10.23	62.50	10.48	8.26	43.48
4	9.47	10.70	40.00	11.85	9.64	37.50	10.70	8.39	34.78
5	11.20	11.62	46.67	12.75	9.82	43.48	12.00	9.28	43.48
6	8.53	9.62	33.33	9.52	10.30	62.50	9.04	8.19	30.44
7	9.60	10.00	40.00	11.55	11.32	62.50	10.61	9.22	43.48
8	9.60	11.28	40.00	11.89	9.39	37.50	10.78	8.20	34.78
9	9.20	9.32	40.00	12.94	10.81	50.00	11.13	8.48	43.48
10	8.27	9.58	40.00	10.61	9.91	39.13	9.48	7.69	39.13
11	11.33	9.55	40.00	14.65	9.87	39.13	13.04	7.91	39.13
12	11.87	10.20	40.00	13.98	9.94	50.00	12.96	7.98	34.78
13	9.33	10.78	40.00	12.65	9.53	37.50	11.04	8.34	34.78
14	9.07	11.17	46.67	11.72	10.07	43.48	10.44	8.61	43.48
15	9.73	10.01	33.33	11.77	10.40	50.00	10.78	8.93	39.13
16	9.87	11.29	46.67	12.32	10.55	62.50	11.13	8.75	39.13
17	9.07	9.97	40.00	11.89	10.47	50.00	10.52	8.66	43.48
18	7.87	9.40	40.00	13.04	10.68	50.00	9.48	7.79	30.44
19	10.53	9.72	40.00	13.04	10.68	50.00	11.83	8.24	39.13
20	9.60	11.28	40.00	10.54	9.73	39.13	10.09	8.49	39.13
21	10.93	9.69	40.00	14.35	10.14	50.00	12.70	7.85	43.48
22	8.27	10.22	40.00	10.28	9.88	39.13	9.30	8.52	39.13
23	10.13	11.45	46.67	14.59	10.83	50.00	14.59	10.83	50.00
24	8.27	9.95	40.00	10.95	8.34	37.50	9.65	7.57	34.78
25	8.40	9.88	46.67	11.50	10.49	50.00	10.00	8.44	39.13
26	8.80	8.77	33.33	11.63	9.56	50.00	10.26	7.75	39.13
27	10.27	10.36	40.00	13.46	11.61	62.50	11.91	9.08	43.48
28	9.60	10.00	40.00	11.89	9.60	37.50	10.78	7.92	34.78
29	6.93	8.51	40.00	10.35	8.69	39.13	8.70	7.19	39.13
30	8.67	9.64	40.00	11.08	10.56	50.00	9.91	8.52	43.48
31	8.13	8.66	40.00	12.42	10.74	62.50	10.35	7.90	39.13
32	8.53	9.43	33.33	12.22	10.16	50.00	10.44	8.00	30.44
33	8.40	9.02	33.33	13.02	10.67	50.00	10.78	8.06	39.13

[7] In Tables 6 and 7, the first number (before the colon) represents the number of context units and the second (after the colon) represents the hidden connected layer.

ad-hoc network, for almost every context setting. The average error is between 6.1% and 7.9%, which is only in four cases worse than the 7.5% shown by the ad hoc network. There is no perceivable relationship between the errors and the decision to connect the context layer with the first or the second hidden layer.

Furthermore, there is an improvement in the percentage of misdefault prediction, as in almost all cases the misdefault percentage is lower than 9.5%, which is the advantage of the ad-hoc network. This is important as it gives further reduction to the misdefault errors, an error class that is of particular interest for banks.

The same result holds when comparing the standard Elman net with the standard Backpropagation, previously shown. Here the overall error ranges between 8.7% and 13.04%, which is better than the [11.57% 14.43%] attained by the former standard net.

6 Discussion of Results

A first consideration that comes to the reader's attention when comparing Tables 6 and 7 with Tables 4 and 5 is that, for the instance at hand, feed-forward networks (both standard and ad-hoc used in Angelini et al. (2008)) are out-performed by Elman nets built just adding a context layer to their feed-forward *ancestors* (also referred to as their *old counterparts*).

Considering first the differences arising in the two *ad-hoc* networks, we can see that the Elman net shows better bonis and default classifications. Of course, a fair comparison is not possible, as Angelini et al. (2008) uses just one topology for this network structure. Nonetheless, numbers reported are useful to give a hint about the performance of both feed-forward and recurrent networks. The Elman-ad-hoc network attains lower valued in misbonis errors than for misdefault errors, but compared with the ad-hoc network, the mean default-mis-classification is always lower (except in 6:1 case), and the same holds for the Standard deviation (Std), which means that results are more stable. Misbonis classifications are always better than Angelini et al. (2008), except for 4 cases, and the mean overall errors are comparable. Please notice that in Tables 6 and 7 we did not insert columns about the minimum achieved error over misbonis, misdefault and overall. This is due to the fact that we found a configuration with null error over the test set for each case, that means better results than the ones reported in Table 1. Furthermore, the results report the superiority, in terms of stability, when compared to Angelini et al. (2008), and a first proof in this direction is shown observing differences between maximum misbo and maximum misdef over the Elman ad-hoc network: for each configuration the maximum achieved value is not as high as one would have expected, given that defaults are notoriously more difficult to classify (with the exception of case 6:1, where the difference is 45%).[8] This also holds for Elman standard networks, and

[8] Indeed, in 8 cases out of 22, that the maximum misbonis is greater than the maximum misdef.

clearly demonstrates the good networks' generalisation capability. Besides, we notice that, in 5 out of 22 cases, the Elman ad Hoc's misbonis mean is greater than the old's one, in just one case its misbonis mean is greater than the old's and, eventually, in 8 cases the old overall value outperforms the new one.[9]

Regarding the standard network, we must consider that Angelini et al. (2008) take into account several network topologies. For this reason, when comparing our current results with theirs, we take into account their *best* results over the object of comparison. Firstly, just in 7 cases (out of 22) the overall performances are worse than their best configuration: this means that we have global performances that are generally better than their old counterparts, with Std of comparable magnitude. Then, in 17 cases we have better misbonis performances, while in 13 cases misbonis performances are worse than their old counterparts: about misdefault, Elman nets improve performances, but not as much as the *Ad-hoc* networks (or the standard's misbonis). With respect to misdefault, old and new performances are comparable, but also here, as seen for the *Ad-hoc* network, the discrepancy between maximum misdefault and misbonis values is reasonable, witnessing the robustness of the approach. Eventually, one should consider that we are targeting against the *best* performances attained by Angelini et al. (2008), meaning that this analysis must be seen as an upper bound over mean differences: Elman's nets always perform better than their ancestors.

It is of particular interest that, for recurrent networks, the *Ad Hoc Elman* performs always better than the *Standard Elman* over both error classes. Thus, there is no need to jointly use the two techniques to tackle one of the two classes of the problem (misbonis or misdef). So, the *Ad Hoc Elman* can be used as a stand-alone method for credit-risk assessment. This is important as Angelini et al. (2008) thought of the two feed-forward counterparts as complimentary (specifically, *Ad-hoc* had very good overall performances but it did not manage to correctly classify default firms, while the *Standard* one had worse overall performances but it succeded in correctly classifying default firms); in our experiments we see both approaches provide us with null test set error as their best performances, but the *Ad Hoc Elman* net overperforms the *Standard Elman*. Summarising, both of them overperform their ancestors, showing that RNN are suitable for credit risk classification.

7 Conclusions

The Basel II Accord on Banking Supervision legislates the framework for banks credit risk assessment. Under this framework banks are required to detain sufficient capital to ensured with 99.9% confidence that losses from unexpected defaults will be covered. Unexpected Loss (UL) is a function of several parameters including probability of default (PD), the likelihood that borrowers will not fulfil their lending

[9] For this comparison, we did not account for tiny differences due to the diverse machine precision. Furthermore, on our rather small sample, these discrepancies are negligible.

obligations in the next 12 months. Banks are required under both the 'Foundation IRB' and the 'Advanced IRB' approach to internally estimate their borrowers' PD.

A number of techniques can be applied to calculate the PD of a borrower. Among them is the statistical credit scoring model. Its derivation is subject to a diverse range of risk factors and their weights. For credit scoring model and weight selection machine-driven techniques, like Neural Networks(NN), can be applied. NN can serve more as default advisors (default–non default) rather than tools for the exact estimation of default likelihood.

In this work we have shown that Recurrent Neural Networks are an effective tool to support lending decisions: their performances can be viewed as state-of-the-art in terms of both best results and robustness. Furthermore, they are able to learn the concept of default and correctly classify default firms: this is of the utmost importance as false negatives (in our case, *misdefaults*) are usually much more important in real world cases. We have furthermore shown that feed-forward NN's performances can be improved by adding a context layer, so modifying the topology of the net without much effort. It is not clear if this assertion can be taken as a general rule or if it depends on the features of the instance at hand. Further research will concentrate this issue.

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Part IV
Modeling from Physics

From Chemical Kinetics to Models of Acquisition of Information: On the Importance of the Rate of Acquisition of Information

Guglielmo Monaco

Abstract. We propose to describe the acquistion of information by the methods of chemical kinetics, with information and knowledge likened to concentrations of chemical species. The Verhulst-like model proposed is characterized by a limiting knowledge which is specific for a given student. The assumption that time and amount of knowledge are continuous variables is more questionable than in chemical kinetics. Consideration of discrete time intervals lead to a quadratic map, showing bifurcation and chaos. The case of two interacting students is also briefly considered. The examples presented show that manageable teaching should be performed at a sufficiently small rate.

1 Introduction

Education systems are typical complex systems, showing many levels of complexity, from the interaction of national councils with educative institutions, to the interactions of school administrators with teachers and students, to the interactions teachers-students, down to the fundamental interaction student-subject to learn.[1] Different levels are strongly interacting, due to the exchange of information and people. A part from the obvious teaching process, information are exchanged through the many channels of either formal or informal evaluation. The exchange of people, in turn, is realized by the students which enter society and by the entry of new personnel in the educative institutions. Educational policies are generally intended

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[1] An introduction to the complex approach in education can be found in Reigeluth CM (1993), Int. J. Ed. Res. 19:117-131; See also http://ccaerasig.com/papers/04/Reigeluth-Chaos2004.pdf. Many more information can be found in the journal Complicity: An International Journal of Complexity and Education.

to increase the quality or also the number of trained people keeping the public expense as small as possible. These policies are object of major social and political concern and this is not surprising if one considers the difficulty of foretelling the behaviour of these complex systems. Optimal choices are here a challenge as the systems are pulled in different directions by the many internal and external forces acting on them. A typical case is autonomy, which is beneficial as far as central evaluation prevents any opportunistic behaviour of local institutions (Woessmann, 2006). Agent based modelling could be a useful tool in this field (Gross and Strand, 2000), but great care must be taken when defining the behaviour of agents. Agents are human beings and could themselves have a complex behaviour. The intrinsic complexity of man can play a role in many social system, and this is why the inclusion of cognitive architectures in social modeling is expected to be very helpful in social modeling (Sun, 2007). The complexity of agents should be particularly relevant in learning, which is often associated to a high emotional involvement.

Models of learning must face the following two hard problems.

1. The process of learning is very subtle and complex. As a familiar example, one can consider that after thousands of years writing has been discovered by mankind, pupils learn writing following different educative styles around the world. It is not obvious whether an optimal educative style can exist, though it seems more reasonable that any given style can be optimized. Different fields are involved in this optimization, from the traditional pedagogy, to the somewhat newer psychology, up to many more fields often grouped as cognitive sciences. Despite the many efforts of the researchers of the different fields, to date, we are certainly very far from a mechanistic description of learning in man, not to say of the positive and negative interactions educatees can have among themselves.
2. It is mandatory to evaluate learning, because the lack of a proper evaluation prevents the validation of any model of learning. Refined statistical techniques have been developed to render the evaluation as objective as possible.[2] However, not all mind features can be evaluated with the same ease. Specifically, the assessment of creativity is far more complicated than the evaluation of basic knowledge.

Here, we intend to discuss models for the acquisition of information, which, though a fundamental step of learning, does not require the involvement of creativity, which is harder to understand and evaluate. We will assume that both the information I and the amount of information acquired (hereafter knowledge K) can be quantified unambiguously, and we will only be concerned with problem 1.

A proper understanding of the mechanisms of acquisition of information would be of great social and economical relevance, as it could lead to optimal designs of syllabi and tracks. In effect the scheduling of the course is a major concern of teacher's work. Not less important are the implications for lifelong learning. The dramatic reduction of the time needed to get information on a given topic thanks

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[2] The theory of specific objectivity is generally considered the prime tool for quantitative evaluation. A rich source of information is www.rasch.org.

to the benefits of communication technology could lead the reader to the illusion of a linear scaling of the learning time: if the information is well organized, its storage is ultimately limited only by the time devoted to the matter. The current success of internet hyperpages, which offer the possibility to deepen this or that on a given page according to the will of the reader, probably follows that idea. However, the greed of information and the resulting overwhelming amount of information are probably the best enemies of information itself. People speaking very fast and saying too many (even interesting) things are not necessarily more communicative than people speaking more slowly. Everyday experience clearly tells that learning is limited in the amount of information that can be acquired in a given time. Learning models must include appropriate mechanisms for this limitation. This point will be addressed in the following.

2 The Evolution of Student's State with Learning

As a dynamical process, learning must change the *state* of the student. In the non-trivial attempt to define such a state, we will first consider what happens in physics.

In classical physics the state of a system of elementary particles is defined by the properties of the isolated particles: masses, charges, positions and velocities. The time evolution of the system, consisting in the the variation of some particle properties, is determined by the potential of interaction of the particles. Depending on the limitations of the observation time and on the size of the system, the description of the system in terms of all the properties of all of its elementary particles can become impossible. This is the case of strongly interacting particles which undergo a complex individual movement in a much shorter timescale than observation; these interacting particles can be described as novel bigger particles moving at observable speeds. This happens in chemistry where, the rapid motion of subnuclear particles is irrelevant for the comparatively slow chemical reactions, characterized by rearreangements of nuclear positions. Similarly the diffusive properties of low pressure molecular gases can be understood in terms of movements of point masses, irrespective of the movement the atoms undergo in the molecule, and more examples could easily be given. In a system with an irresolvable complexity (a true *system* according to its veritable ethymological meaning), no bigger particles can be defined and one is forced to consider the system as a whole. This is the case of thermodynamics, where states are defined by a few properties, which are averages over all of their many particles.

Thus, whenever a group of particles cannot be followed individually, positions and velocities cannot be measured, and in the definition of state they are substituted by the smallest set of collective properties. On the other hand, the limited observation time can even simplify the description of the evolution of the system. Indeed, during the observation, the time variation of certain properties can be negligible within desired precision. The slow varying properties can be considered as parameters when considering the evolution of the fast varying properties. This is the situation in electronic spectroscopy. Due to their heavier mass, nuclei are much slower than electrons and, on the time scale of some spectroscopic measurements, nuclei can be considered fixed and act as parameters in determining the properties of the electrons which behave as a whole (the electronic state). Interestingly, albeit a molecule is clearly a true *system*, where a change in the position of any particle is fundamentally linked to a variation of the positions of all the other particles, the different timescales enable to consider the evolution of the properties of a subset of particles (the electrons), while other particles are kept as parameters (the nuclei).

The student is certainly a complex system, which must be treated as a whole, and the question arises: can a set $\{\mathcal{P}\}$ of relevant properties (for the education process) be identified for him (and the teachers)? To have an operative definition, the properties must be accessible: they should be susceptible of evaluation. This suggest to consider the objectives of learning, which have been thouroughly considered by Bloom (Bloom, 1956), and successively widely applied (Anderson and Krathwohl, 2001).

The idea that the whole student can be defined by a set of relevant properties is no more than an assumption and we think that the mental state ($\mathcal{S}$) of the student must also be considered: the same student can perform very differently depending on his mental state and a better performance can stem not only from his superior knowledge or capabilities, but also from a more frequent occupation of a positive mental state. Although such a mental state cannot be determined in any easy way, its very existence is routinely assumed in teacher's work. In this framework we can indicate as $|\mathcal{S}, \mathcal{P}\rangle$ a student in mental state $\mathcal{S}$ with properties $\mathcal{P}$. A model of learning should follow the time evolution of the valuable properties, considering that this evolution depend on the mental state of the student. Preliminary explorations of the possibility to model acquisition of knowledge and maturity considering two different mental states have been reported elsewhere (Monaco, 2008). The demanding requirement of considering the time evolution of the mental state can be simplified if one considers the overall perfomance of a large set of non-interacting students, where some averaged mental state matters, or a single student which has a constant mental state during learning. The latter limiting case is probably more likely for low level of education, focused on acquisition of knowledge, rather than critical thinking. This is the case of the present paper, which is focused on the acquisition of knowledge K.

How to describe the time evolution of K? Cognitive sciences are revealing the complex processes that occur in our brain during learning. The current opinion is that all brain processes are ultimately electric discharges between neurons, possibly mediated by neurotransmitters. The electric discharges are consequences of a change of concentration of positive and negative ions inside and outside neurons. Novel stimuli – such as the incoming information during learning – are a cause of the change of the distribution of ions. The temporary changes in the concentration of ions can lead to permanent signatures in the brain. Experiments on mussels (Bayley and Kandel, 2008) and mice (Dobrossy et al., 2008) have shown that memory – the retention of information – is connected to synaptic remodeling and synaptic growth

(Bayley and Kandel, 2008), and to neurogenesis (Dobrossy et al., 2008). In practice changes of the concentration of ions are followed by changes of the concentration of synapses or even neurons. Therefore, according to this picture, it is well tenable the attempt of describing both I and K as concentrations and their evolution in terms of standard processes of chemical kinetics.

3 A Verhulst-like Model

The picture of learning emerging from neuroscience is compatible with the basic tenets of cognitivism, which assess that in order to learn something it is necessary to find some correspondence between former knowledge and the new information. The requirement of this correspondence will generally differentiate correlated pieces of information, when they are acquired successively, even if they bear the same amount of technical information. From classroom practice, one often has that a good knowledge of first topics of a basic course speeds up considerably the acquisition of successive ones. On this ground one can imagine to describe the process of acquisition of information I as a typical autocatalytic chemical process:

$$K + I \xrightarrow{k_1} 2K \,, \tag{1}$$

where k_1 is a rate constant. If process 1 is assumed as an elementary chemical process, the rate of change of K would be simply

$$\frac{\mathrm{d}K}{\mathrm{d}t} = k_1 K I \,. \tag{2}$$

This Malthusian-like increase will be certainly limited for high amount of information acquired. One can try to account for this limitation including the possibility that different pieces of knowledge annihilate as a result of confusion. Confusion, in turn can be generated either by information not related to the information to be acquired I, or by that same information I:

$$K + K \xrightarrow{k_2} \tag{3}$$

$$K + K \xrightarrow{k_3 I} \,. \tag{4}$$

If processes 1, 3 and 4 are considered together as standard kinetic equations, one ends up with

$$\frac{\mathrm{d}K}{\mathrm{d}t} = k_1 K I - 2\left(k_2 + k_3 I\right) K^2 \,. \tag{5}$$

For a constant amount of information, Eq. 5 has a limit non-zero knowledge

$$K_\infty = \frac{k_1 I}{2\left(k_2 + k_3 I\right)} \,.$$

Rewritten in terms of K_∞, Eq. 5 becomes

$$\frac{\mathrm{d}x}{\mathrm{d}t} = rx(1-x)\,, \tag{6}$$

where $x = K/K_\infty$ is a normalized knowledge and $r = k_1 I$ is the initial rate of acquisition of knowledge. Eq. 6 is the Verhulst logistic equation, originally developed to describe a population growth limited by a death process and ending in a finite limiting population, in contrast with the unbound Malthusian growth.

From the solution of Eq. 5,

$$K(I,t) = \frac{K_\infty K_0}{K_0 + (K_\infty - K_0)\,\mathrm{e}^{-rt}}\,,$$

one obtains that knowledge increases or decreases with both amount of information and time, depending on whether the starting knowledge K_0 is either lower or greater than the limit knowledge K_∞. Thus, provided that the limiting knowledge is not yet achieved ($K_0 < K_\infty$) the educatee will benefit from more time devoted to learn as well as from a higher amount of information. Nevertheless a beneficial effect of an increase of information can also occur for $K_0 > K_\infty$, if the increase is substantial as shown in Fig. 1.

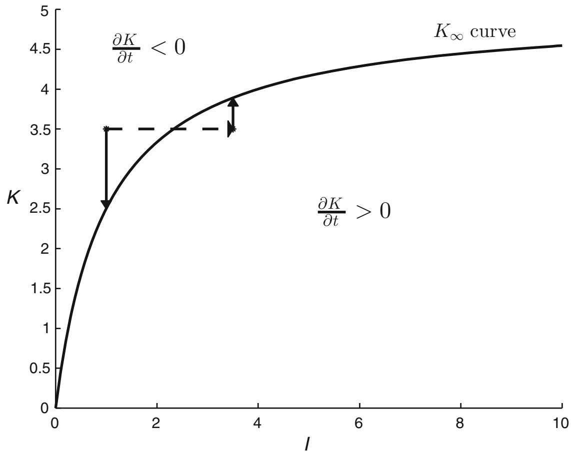

Fig. 1 The curve of limiting knowledge K_∞ as a function of the information to be acquired I. Starting from a knowledge K_0 above (or below) the curve, knowledge decreases (increases) in time (*arrows with continuous line*). In the *upper region* both partial derivatives $(\partial K/\partial t)_I$ and $(\partial K/\partial I)_t$ are less than zero. However a finite increase of information (*arrow with dashed line*) can lead to a substantial increase of limiting knowledge

3.1 Some Limitations of the Model

The Verhulst-like model is certainly more realistic than the Malthusian exponential growth. Moreover it differs considerably from the linear scaling of the learning time, which assumes that all students can achieve the same results, albeit in different times. Indeed in the Verhulst-like model students with different personal kinetic parameters k_1, k_2 and k_3 will have a different limiting knowledge. Clearly the personal kinetic parameters can be expected to evolve together with the person; nevertheless, if this evolution is slow with respect to the teaching process – as often happens –, less talented students cannot be expected to perform as the more talented ones even if they study for a longer time.

Personal evolution, resulting in time-dependent kinetic parameters, is only one of the limitations of the above model. A stronger limitation is the assumption that the acquisition of knowledge can be likened to an *elementary* chemical process. In an elementary chemical process reactants interact to lead the products directly. Most chemical processes are instead complex: the transformation of reactants into products occurs by many elementary processes, which sometimes are unaccessible by experiments and can only be hypothesized as possible interpretations of the overall rate. If process 1 is likened to a complex chemical reaction, its rate is expected to show different behaviours in different intervals of K and I. Following the customary approach of chemical kinetics, the rate in a given interval can be approximated by

$$\frac{\mathrm{d}K}{\mathrm{d}t} = k_1 K^k I^i ,$$

where k_1 and the exponents k ands i (the orders of reaction) depend on the interval of K and I. Acquisition of knowledge is better described by a complex reaction than by an elementary process. Indeed man has a limited capability of perception of stimuli: the reading rate of a sentence is not influenced from the following one. Therefore, for a sufficient amount of information the rate of acquisition of knowledge is expected to be independent of the information itself: $i = 0$. In this case the curve of the limiting knowledge will be a decreasing function of information amount. Finally, if the information-driven confusion is also neglected the curve of limiting knowledge would reduce to a horizontal line. Probably none of these very different behaviours is totally unsound and each of them could be valid for different sets of personal parameters and teaching environments.

Last but not least limitation concerns the timescale. A single reaction has its own dynamics and cannot occur in a time shorter than a given Δt, generally less than 10^{-12} s. Chemical kinetics deals with very large numbers of molecules, tipically of the order of 10^{23}. In typical slow reactions billions of billions of molecules react in a second and the kinetic constants are averages over this enormous amount of events. It is therefore not suprising that the timing of the single chemical reaction and the discrete amount of molecules are irrelevant on the whole. Indeed classical chemical kinetics are determined by differential equations, defined in terms of infinitesimal increments of concentrations and time.

The situation of learning is substantially different for three concurrent changes:

1. the timescale of human perception is limited by the duration of the electric spikes occurring in synapses, of the duration of roughly 10^{-3} s;
2. in a couple of seconds no more than roughly 7 elements of information can be acquired by man in the short-term memory (Miller, 1956);
3. the numbers of neurons and synapses in a human brain, roughly 10^{11} and 10^{14} (Braitenberg V and Schüz, 1998, Tang et al., 2001), are smaller by many order of magnitude than the typical number of molecules in a chemical reaction.

Clearly the assumption that K, I and t can be considered as continuous variables is less founded than the analogous assumption made in chemical kinetics.

4 Finite Time Intervals

As a first step in the direction of accounting for the limitations of the continuous Verhulst-like model, we will consider the effect of a finite Δt.

4.1 A Discrete-time Verhulst-like Model for a Single Student

For a finite Δt equation 6 can be approached by finite differences:

$$x_n = x_{n-1}(1 + a - a x_{n-1}), \qquad (7)$$

where the parameter $a = k_1 I \Delta t$ is the amount of information supplied in a time Δt, weighted by a student-dependent kinetic parameter k_1: a rate of acquisition of information. Equation 7 is a quadratic map, and is very close to the well known logistic map (May, 1976).

The value of x_n cannot be negative, as this would indicate an unacceptable negative knowledge. According to Eq. 7 this situation happens if $x_{n-1} > 1 + a^{-1}$. On the occurrence of this situation we set $x_n = 0$. The limiting values for x_n ($n > 500$) starting from $x_0 = 0.01$ are reported in Fig. 2.

For $a < 2$, the knowledge approaches its Verhulst like limit ($x_\infty = 1$), although with different times (for small a the approach of the limiting value is extremely slow). For higher values of a successive x_n values can oscillate between two ore more different values, either higher or lower than the Verhulst-like limit. Moreover for certain values of the amount of information, the variation of x_n's has no periodicity at all (deterministic chaos). Above $a = 3$ the limiting normalized knowledge vanishes.

Thus, according to this model, the system student-information has a rate of acquisition of information, $a = 2$, that should not be overcome in order to prevent a chaotic and thus unmanageable behaviour. If bistability occurs supplying an amount of information I^* in a time t^*, a 150% increase of the amount of information ($1.5I^*$) results in complete loss of learning capability.

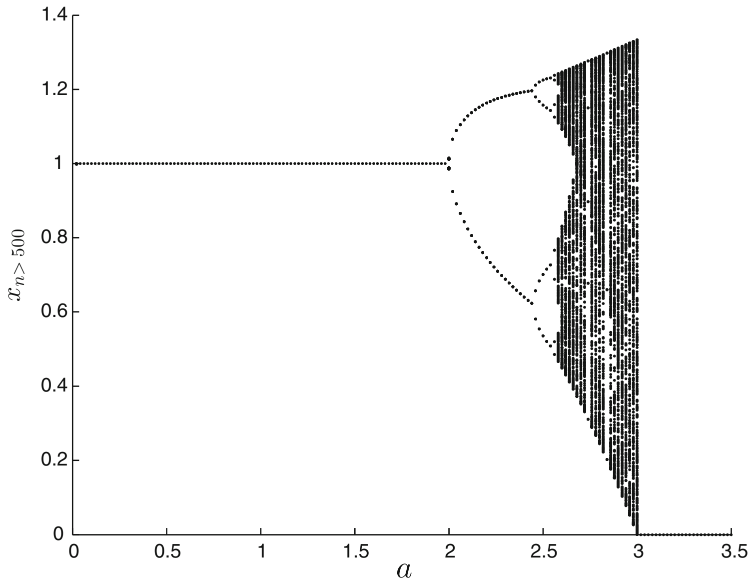

Fig. 2 Limiting values x_n of normalized knowledge as a function of the rate of acquisition of information a. x_0 was set to 0.01

4.2 A Discrete-time Verhulst-like Model for Two Students

As a final example of the complex pattern that can be expected for the acquisition of knowledge we will consider two interacting students with same personal kinetic parameters, grouped in parameter a. The interaction is a further source of uncertainty because neither the functional form of the interaction nor its parameters can be assessed a priori. We will consider a model in which the knowledge of a student can act like a source of information for the other one. In these case the two normalized knowledges will be given by

$$x_n = (1+a)x_{n-1} - ax_{n-1}^2 + c_{\text{int}}x_{n-1}y_{n-1} ,$$
$$y_n = c_{\text{int}}x_{n-1}y_{n-1} + (1+a)y_{n-1} - ay_{n-1}^2 ,$$

which are simplified forms of the equations used to study strange attractors. Figures 3 and 4 report two different cases for the interacting students. In Fig. 3 we see that small interaction parameters ($c_{\text{int}} = 0.1$ or $c_{\text{int}} = -0.1$) for students starting from the same prior knowledge ($x_1 = y_1 = 0.01$) results in a limiting normalized knowledge either higher or lower than the one expecxted in absence of interaction, but the overall behaviour of the x_n/a map is substantially unchanged.

A different situation is obtained if the two students have a different prior knowledge ($5x_1 = y_1 = 0.05$). In this case we get that i) for small rates the limiting knowledge is increased, ii) the onset of the unmanageable regime occurs earlier

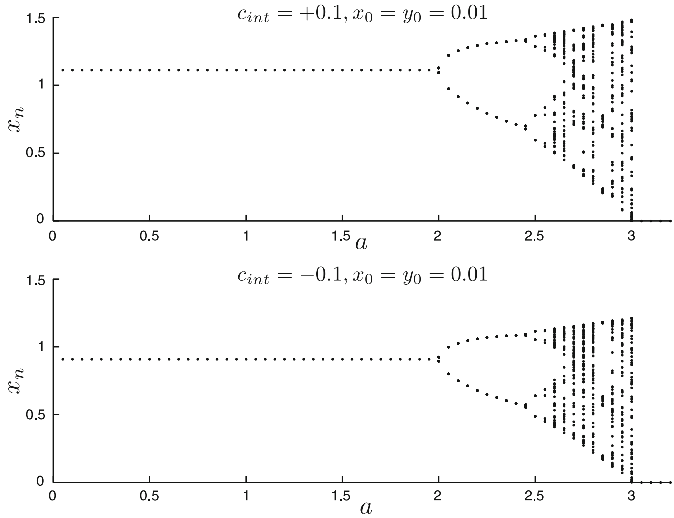

Fig. 3 Limiting values of normalized knowledge x_n ($n > 500$) for two interacting students with the same value of the parameter $a = k_1 I \Delta t$. Starting knowledge and interaction parameters have been chosen as indicated on the graphs. In these cases, the normalized knowledge y_n's of the seconds student are equal to the x_n's

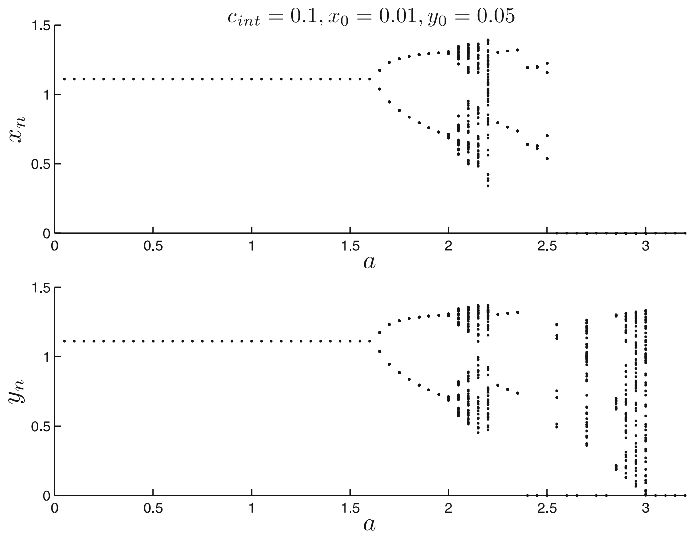

Fig. 4 Limiting values of normalized knowledge x_n and y_n ($n > 500$) for two interacting students with the same value of the parameter $a = k_1 I \Delta t$. Starting knowledge and interaction parameters have been chosen as indicated on the graphs

than in the case of the single student, iii) the worse starting student goes earlier to zero knowledge, iv) both sudents have two zones of bistability, v) the better starting student has rates leading to zero knowledge inside the unmanageable region.

The modifications occurred are a clear example that the learning maps can become considerably more complicated on account of interaction.

5 Conclusions

Agent based modeling has been applied successfully to a variety of social systems. Can it be applied to education as well? Can agent-based modeling help in designing sillabi and tracks for a class of interacting students? A positive answer would mean that, given certain rules for the acquisition of knowledge and for the interactions among students, one can recover the best set of teaching parameters, among which there is the rate of acquisition of information. For too much complicated rules, models would soon become useless as it would be difficult to separate the effect of the performance of a single student from the rules of interaction among students.

Here, we have investigated a learning model based on the analogy of knowledge with the concentration of a chemical reactant, as suggested by recent insights of cognitive sciences. The Verhulst-like model proposed is characterized by a limiting knowledge which is specific for a given student. The assumption that time and amount of knowledge are continuous variables is more questionable than in chemical kinetics. This consideration has led us to consider discrete time intervals, which lead to a quadratic map, showing bifurcation and chaos. This complex behaviour is even enhanced upon consideration of two interacting students. According to these models a teaching process, susceptible of optimization through agent based modeling can only be expected when the rate of acquisition of knowledge is sufficiently low.

Acknowledgements The author dedicates this contribution to the memory of the great man and thinker Prof. G. Del Re, who first suggested him to consider education in the light of the theory of complex systems.

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Thermodynamic-like Approach to Complexity of the Financial Market (in the Light of the Present Financial Crises)

Andrei Khrennikov

Abstract. We consider dynamics of financial markets as dynamics of expectations of its agents and discuss such a dynamics from the point of view of phenomenological thermodynamics. We describe a financial thermodynamic-like cycle and the financial analogue of a heat machine. We compare complexity of the financial cycles with complexity of analogous thermodynamic cycles. Our thermodynamic-like model of the financial market induces a rather unusual interpretation of the role of financial crises. In contrast to the common point of view, financial crises play a crucial role in functioning of the modern financial market. A financial cycle could not be completed without such a stage as well as any (physical) thermodynamic cycle. Thus, in spite of its destructive (at the first sight) consequences, the stage of financial crises is as well important as the stage of "boiling of the financial market". We also discuss a possible decision-making strategy for traders induced by our thermodynamic model. It is, in fact, a decision strategy at the market with an arbitrage possibility for a special group of traders.

1 Introduction

Financial market is a gigantic system operating with huge ensembles of securities. It is natural to expect that its functioning has common features with the functioning of physical systems, which operate with enormous statistical ensembles. One can expect methods of classical statistical physics, together with methods of classical thermodynamics, to work successfully in financial processes modeling. This model class is the most important component of econophysics (Mantegna and Stanley, 1999). Chief contribution of the author to econophysics, see Khrennikov (2000,

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2002, 2004), is the realization that a financial market is an information system, operating with information values, cf. Soros[1], 1987. A similar idea (in various forms) was present in publications of other authors, Choustova (2002, 2004, 2006, 2008); Haven (2006); Arouri et al. (2009); Purica (2009); Haven and Khrennikov (2009). The key part is played not only by market's internal information media, e.g., shares, but also by psychological values, for example, expectations of market players, news and prognoses, that are published in newspapers and announced on TV, politicians' claims and so on. The latter are immediately transformed into modified expectations. The expectation variable x will play a fundamental role in our model. The simplest way to quantify x is to order the degree of optimism vs pessimism and encode it by real numbers, $x \in \boldsymbol{R}$, where the symbol $\boldsymbol{R}$ denotes the real line.

Thus complexity of the financial market cannot be simply reduced to complexity of actions of traders and dynamics of real economics. Extremely complex system of social relations coupled to mass-media and even politics plays an important role in the dynamics of psychological states of agents of the financial market by influencing their expectations with respect to future development of stock prices. Consider possible dynamics for the x-variable, say

$$\frac{\mathrm{d}x}{\mathrm{d}t}(t) = f(x, y), \text{ where } y(t) = (m(t), e(t), n(t)) \;, \tag{1}$$

the m-variable describes parameters of market's activity, i.e, Dow Jones Index, the e-variable describes the state of real economcs. i.e., GDP real growth rate, the n-variable describes information provided by mass media and state authorities, Central Banks (CBs) or Federal Reserve System. Taking into account the n-variable is a crucial point of our model. The main problem is that quantification of n is not a simple task. For example, n contains Federal Funds Rate as one of its coordinates and this coordinate is easily quantified. However, it is not easy to quantify the information impact of say an interview of a politician or a top-level financial expert. In a mathematical model it is natural to consider n as a random variable, $n = n(t, \omega)$, where ω is a random parameter. Following tradition of the financial mathematics, we can consider n is driven by a stochastic differential equation (SDE), say of the diffusion type:

$$\mathrm{d}n(t,\omega) = a\,(t, n(t,\omega), \omega)\,\mathrm{d}t + b\,(t, n(t,\omega), \omega)\,\mathrm{d}w(t,\omega)\,, \tag{2}$$

where $w(t, \omega)$ is the Winer process, so $\mathrm{d}w(t, \omega)$ describes informational noise coming (to market's agents) from mass-media, finanical and state authorities, experts. The coefficient $a(t, n, \omega)$ describes the main trend in mentioned information, the coefficient $b(t, n, \omega)$ describes "diffusion of information". In coming model, the monitoring of $a(t, n, \omega)$ provides a possibility of monitoring of the n-variable and, hence, the x-variable, expectations of market's agents. In contrast to (2), dynamics of expectations (1) does not contain a diffusion-type term. In the ideal case (which

[1] G. Soros was one of the firsts who started to treat (both practically and theoretically) the stock market as an information system which is sufficiently independent from the dynamics of the real economics.

we would like to emphasize in our model), the main source of randomness is the "news"-variable $n(t, \omega)$. Of course, the real dynamics of expectations also should be described by SDE:

$$dx(t, \omega) = f(t, x(t, \omega), y(t, \omega), \omega)dt + \sigma(t, x(t, \omega), y(t, \omega), \omega)\, dw(t, \omega) . \quad (3)$$

The second term describes randomness of dynamics of expectations. However, in our model we will pay the main attention to monitoring of $x(t)$ through monitoring of $n(t)$. The impact of $\sigma(t, x(t, \omega), y(t, \omega), \omega)dw(t, \omega)$ can be neglected, not because it is negligibly small, but because at some stages of the evolution of the finanical market its contribution can be neglected comparing with the contribution of the "news"-variable $n(t, \omega)$.

At the moment (as a consequence of the recent crash) the problem of regulations at the stock market is a hot topic for debates in financial, state, and academic circles. However, typically solutions of this problem are reduced to regulations in the form of more rigid rules for various operations, bonus system and so on. In such regulatory projects the real complexity of the stock market (which includes psychological states of people involved in operations with financial assets) is not taken into account. Our model emphasizes a role of psychological behavior of market's agents, i.e., the variable $x(t, \omega)$. The main message of this model to financial and state authorities is that practically meaningless to appeal to rigid regulations for financial operations, to modifications of the bonus system and so on. It would not change crucially the basic structure of financial crises – a possibility for arbitrage for a special group of people. (This point will be clarified little bit later). Only open recognition of such a possibility of "selected arbitrage" will change crucially market's psychology and it will restrict the rick-oriented behavior of traders. It is clear that a decision-making strategy under context of arbitrage (even for a special group of traders) will differ crucially from the present risky strategy. Later we will not use so to say "individual state" variables, x and y. Therefore it is a good place to compare our approach with traditional financial mathematics. Coming back so far as to pioneer Bachelier's model, we get a diffusion-type equation for the price change variable v:

$$dv(t, \omega) = a_B(t, v(t, \omega), \omega)\, dt + b_B(t, v(t, \omega), \omega)\, dw(t, \omega) . \quad (4)$$

This equation has no direct coupling with the expectation variable x and, hence, no direct coupling with the the "news"-variable n. Thus the idea of a possibility of monitoring the v-variable through SDEs (2) and (3) is totally foreign to the modern financial mathematics. The latter is based on the efficient market hypothesis which implies the total impossibility of permanent arbitrage at the financial market. The efficient market hypothesis which was formulated in 60th by Samuelson (1965) and Fama (1970) is a cornerstone of practically all modern approaches to modeling of financial processes; in particular, so called financial mathematics based on theory of stochastic processes. Our thermodynamic-like approach rejects the efficient market hypothesis (at least this hypothesis should be used under essential restrictions induced by the real situation at the financial market), (cf. Choustova, 2002, 2004, 2006, 2008).

The second most important moment in creation of our model is the realization that for the prediction of the behavior of stock market, one should use collective, rather than individual, variables. In physics, phase-space coordinates (position and velocity, or to be more precise momentum: (x, p)) of a single gas particle are routinely chosen as individual variables. In principle, we can describe the physical dynamics by means of these variables. Here we have Hamiltonian dynamics or stochastic dynamics of Brownian motion (or of the more general diffusion processes) type. Most financial market models work with this class of variables, application of stochastic processes being the most popular technique, see Shiryaev (1999), or Mantegna and Stanley (1999). Of course, already in physics it has been fruitless to undertake solving a system of Hamiltonian equations for millions of gas molecules.[2] The use of probabilistic description has been proposed. Instead of a system of Hamiltonian equations, we shall use Liouville equation. Describing price fluctuations by means of stochastic processes, one is mostly interested in probabilities dynamics, making use of forward and backward Kolmogorov equations. Yet there is another approach to the collective variables introduction, that is the thermodynamic one. Operating with such variables as gas temperature T, its energy E, work A, volume V, pressure P, we can describe the output of the "activity" of big ensembles of gas molecules. We apply this approach to the financial market. It may be said, that this paper is about *financial thermodynamics*, (cf. von Neumann, 1945; Cimbleris, 1998; McCauley, 2003, 2004; Mimkes, 2006; Smith and Foley, 2009; Petersen et al. 2009). Handling the terms such as the temperature of the market, the energy of the market, work of monetary funds, we create the thermodynamic model of the market. In our model thermodynamic financial variables are defined not only by "hardware" of the market, but by its "software" as well. Not only real economical situation, not just stock prices, but psychological factors as well contribute to, e.g., the temperature of the market, (cf. Soros, 1987; Khrennikov, 2004; Haven, 2006; Choustova, 2002, 2004, 2006, 2008; Arouri et al. 2009; Purica, 2009; Haven and Khrennikov 2009). Thus, our model is informational financial thermodynamics.

Now there is the last important moment. In practically every known econophysical model, financial processes have been considered as objective processes. This point of view is the most distinct in the models based on the theory of stochastic processes. Financial randomness has been considered an objective one Mantegna and Stanley (1999). The simplest form of this postulate, Malkiel (2003), can be found in the financial market model where stock prices considered to be random walking. From this point of view, the play strategy better than the one based on coin flipping (heads – go long, tails – go short) cannot be found. Of course, even the advocates of stock market objective randomness understand this model to be somewhat primitive. They tried to "improve" on it by considering new and new classes of stochastic processes: different modifications of Brownian motion, Levy processes, martingales, submartingales, etc., Shiryaev (1999). For example, geometric Brown-

[2] A hundred years ago it was an unsolvable mathematical problem. Nowadays, one can, again in principle, attempt to compute it; however, visualization of millions of trajectories would be still impossible.

ian motion was very popular during a long period. It satisfies the following SDE:

$$\mathrm{d}P(t,\omega) = uP(t,\omega)\mathrm{d}t + vP(t,\omega)\mathrm{d}w(t,\omega) \tag{5}$$

where u ("the percentage drift") and v ("the percentage volatility") are constants. The equation has an analytic solution:

$$P(t,\omega) = P(0,\omega)\exp\left(\left(u - v^2/2\right)t + vw(t,\omega)\right) . \tag{6}$$

The crucial property of the stochastic process $P(t,\omega)$ is that the random variable

$$\log\left(P(t,\omega)/P(0,\omega)\right)$$

is normally distributed. However, all these exercises in probability theory have been only meant to confirm the postulate of stock dynamics being akin to the stochastic dynamics of gas flow.[3] If one wants to explore our approach in the framework of modern finanical mathematics, then SDE for the price dynamics or price-change dynamics, cf. (4) or (6), should be modified to include the influece of the psychological variable $x(t,\omega)$ and, hence, the news variable $n(t,\omega)$. In the class of SDEs of the diffusion-type, one should work with SDE of the form:

$$\mathrm{d}P(t,\omega) = A\left(t, P(t,\omega), x(t,\omega), \omega\right)\mathrm{d}t + B\left(t, P(t,\omega), x(t,\omega), \omega\right)\mathrm{d}w(t,\omega) , \tag{7}$$

where the psychological state $x(t,\omega)$ is driven by SDEs (2), (3).

Well-known fact however is that people for long have known how to make artificial systems to control hot gas flows. A work is performed by these systems; which, meanwhile, consume fuel. Moreover, as the power of the systems and their number grow, the negative influence of their activity becomes more and more considerable. Financial market functioning studied through thermodynamic analysis bears apparent resemblance to the operation of a heat machine. Thereat, the fact that the machine implementing a financial cycle – which is a direct analogue of a thermodynamic cycle – is manmade cannot but strike one's eye. In principle, financial specialists have been monitoring financial cycles for long, and the records can be easily compared to our financial-thermodynamic description, (cf. Malkiel, 2003).[4] However, econophysics approach makes the structure of financial cycles particularly simple. The analogue to heat machine is the strongest metaphor rendering the shadowy structure given by liberal economics sharp. The main conclusion of this work is: After (or, possibly, simultaneously with) the invention of physical heat machines, man contrived "financial heat machines". Both physical and financial (informational) machines enable work doing (in the latter case the work has the meaning of a profit), realizing cycles of the thermodynamic type. Unlike the

[3] This means that modern financial mathematics has a clear ideological dimension. Financial processes are like natural phenomena. One cannot avert a financial hurricane just as one cannot stop a hurricane formation in Caribbean Sea. There are only aftereffects to fight with.

[4] Though the author of this book promotes random walk model, the book contains amazingly demonstrative description of a number of thermodynamic-like financial cycles.

physical heat machine, financial one can break the second law of thermodynamics, allowing, in principle, for creation of financial perpetuum mobile.[5] And still, both engines' work feature the last stage, that is steam cooling. Financial notion of this stage is financial crisis. Thus, financial crisis in our model is not generated by the objective randomness of the market. Just as in physical heat machine, it is a critical stage of the working cycle. Without it, physical or financial heat machine just cannot work.

Well, financial crisis is not a financial hurricane of a random nature. It is a man-designed stage of "financial heat machine" operation. Crises elimination would wreck the machine and render financial market unattractive to its creators. Apparently, one should not undertake to fight financial crises as such, but rather alleviate the damage to the "environment". Trying to suppress the use of thermodynamic cycles in physics is absurd. No wiser would be to try to forbid the use of the financial thermodynamic-like cycles. Informed people will seek to use the cycle nonetheless. World-wide Counteragency for Financial Heat Machines is a non-starter. That is why we should look to lower the "exhaust", and move to more advanced models.

Besides "justification" of crises as objective stages of market's development, this paper opens the door to a new domain of financial engineering – the use of financial counterparts of thermodynamic theory and improvement of "financial heat machines" by comparing them with the most effective physical heat machines. It is easy to see that thermodynamic-like financial cycles considered in this paper do not matches completely the Carnot cycle (the latter is the most efficient in thermodynamics). It is an interesting, but complicated project to design Carnot-like financial cycles. We remark that the analogy between cyclic functioning of physical heat machines and financial cycles was already explored in Khrennikov (2005). In the present paper we essentially extend this approach: by performing detailed analysis of economic and financial consequences of the thermodynamic-like approach to the financial market and by illustrating our model by the present financial crises. We also consider more complex financial cycles than in Khrennikov (2005). Moreover, a deeper analysis performed in this paper demonstrated that, in contrast to the claim in Khrennikov (2005), thermodynamic-like financial cycles do not coincide with the Carnot cycle (although they have a lot in common), see Sect. 6 for comparison of the physical Carnot cycle and financial cycles under consideration. Another important new contribution is exploring of temperature-entropy diagram in the financial framework. It completes essentially the picture of market's dynamics given by the (V, P)-diagram, where V is the volume (of shares) and P is the price (of per share). Since our approach is based on the dynamics of the psychological states of agents of the market, we define market's entropy on the basis of the probability distribution of variety of expectations of future development of the market. We will not take care of the individual dynamics of the expectation variable, $x(t, \omega)$, nor the news dynamics, $n(t, \omega)$. To define dynamics of e.g. entropy, $S(t, \omega)$, we need only the probability distribution $p(t, x)$ of the stochastic process $x(t, \omega)$.

[5] While the Maxwell's demon does not exist in physics, financial Maxwell's demons may exist (and draw good income, too).

2 Ideal Thermodynamic-like Cycle at the Financial Market

For simplicity, let us consider functioning of the financial heat machine for shares of one fixed company. Suppose that totally it was issued V shares. Suppose that a group of people, say G, "designers of a financial heat machine", was involved in the issue of shares and they got $\Delta V = V - V_1$ hares which they do not sell (for a moment) and the rest of shares V_1 circulates at the market.[6]

We discuss the dynamics of the market when the number of shares, which do not belong to G, is equal to V_1 and the price (at so to say "open market") is equal to P_1. Thus the starting point is characterized by the pair $1 = (V_1, P_1)$.

Then we consider the market dynamics when the expectations of the price grow to $P > P_1$, but (for simplicity) no new shares sold (so people from G just wait and do not sell their shares) and the market arrives at the point $2 = (V_1, P)$. At the moment we do not discuss the method how this rise of expectations is achieved, see Sect. 3.

Then the group G starts to sell shares at the fixed price P, and the aim is to sell all their shares, so that, finally, the total volume of shares at the financial market will be equal to V. For simplicity we assume that the price of shares during this process will be fixed. Suppose that the G-group sold all its shares. The market arrived at the point $3 = (V, P)$.

Then a market crash occurs: the price of shares falls from P to P_1. The market arrives at the point $4 = (V, P_1)$.

Then G-people buy again ΔV shares at the price P_1 reducing volume (at, so to say, "open market") to V_1. Thus the market arrives back at our starting point $1 = (P_1, V_1)$.

What will be the result of all these four processes? Namely, the result of the cycle:

1. Rise of the price $1 = (V_1, P_1) \to 2 = (V_1, P)$.
2. Then G-people sell their shares at the high price $2 = (V_1, P) \to 3 = (V, P)$.
3. Then market crash $3 = (V, P) \to 4 = (V, P_1)$.
4. Finally, G-people buy again shares, but at the low price $4 = (V, P_1) \to 1 = (V_1, P_1)$.

The result will be the profit:

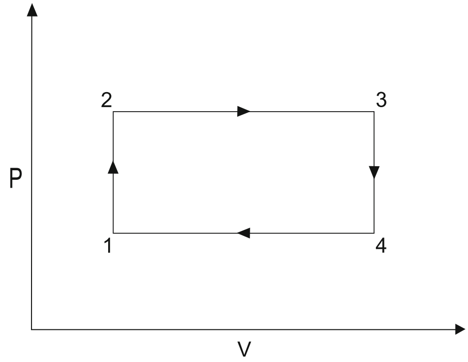
$$A = \Delta P \Delta V \,,$$

where $\Delta P = P - P_1, \Delta V = V - V_1$.

The above described picture is obviously an analog of a cycle in thermodynamics, e.g., the Carnot cycle. The price P is the analog of pressure, and the volume of the market (the number of shares) V is the analog of volume of gas. The profit A is the analog of work (or energy). If we will continue these physical analogies, expectations will be the analog of vapor. We mention that Malkiel (2003) compared expectations of people at the financial market with Castles-in-the-Air.

[6] There are other ways to collect shares at sufficiently low price, see considerations below.

Fig. 1 (V, P)-diagram for the ideal financial cycle



By my model if one would be able to influence the expectations of the people operating at the financial market, then one would be able to make profits using the described financial thermodynamic-liked cycle. We remark that the profit produced by the financial Carnot cycle is given by the area of the circuit 1234 (as for the physical Carnot cycle).

Remark. (On the structure of G-group) The group of people who "design" the concrete financial cycle has quite complicated social structure. Only in the simplest case it can be sharply defined, e.g., John, Bill, Emmanuel. In general it is a diffuse social group and the measure of involvement in the financial thermodynamic-like cycle varies essentially in this group; moreover, it can vary essentially with time. Besides more or less well defined kernel G_0 consisting of people having money, experience, and contacts, at some stages G includes a large group, say G_1, of financial speculators who know that a new cycle has been started. The latter can buy respective financial asset(s) during the stage $(1 \to 2)$ when the price is not yet too high. They can actively participate in heating of the financial market and then in its cooling. They can obtain essential profits from a cycle, but their profits are very small comparing with profits of the G_0-group. In contrast with the G_0-group, G_1-people can be considered as speculators, since they take some risks. However, their risks are very low. They know more or less precisely when this cycle starts and when it will come to crash; if not, they secure their profits by a well developed system of bonuses a nd other similar instruments. Of course, G_0-people cannot be treated as speculators. They take practically no risks, their activity is based on a possibility of arbitrage. The latter is incorporated in the structure of a cycle.

3 A Role of Mass-media, a Financial Perpetuum Mobile

To get a better understanding of this concept let us discuss the standard steam engine[7] from a thermodynamic point of view. In a steam engine we have a boiler,

[7] Functioning of the steam engine matches better with financial cycles of former times, modern financial cycles are closer to the Carnot cycle.

a cooler, and the cylinder, where the thermodynamic cycle (in the idealized case) takes place.

The cylinder in our language is the analog of a financial market. What are the analogues of the boiler and the cooler (without which the steam engine will not work)? The boiler is a component which is used to "heat the expectations" of participants of the market and to increase the price. We propose to use mass media and other sources of financial influence as the analogue of a boiler. If one will be able to manipulate the expectations of market agents independently of the real situation which corresponds to the activities of the company (to cheat market agents), then this will be the analog of a perpetuum mobile. We see, that while in physics a perpetuum mobile is absolutely impossible, in economics such mobile may occur. The role of a cooler will be again played by mass media, which in this case will distribute negative information. The important point here is the possibility of an economic perpetuum mobile for those who can control communications (or for those who control the ones who control communications). While in real economy, of course, one has to put some fuel into the engine (to produce some real goods and services), the reaction of the stock market and the increase of prices of shares can be considerable higher than changes in real productivity. This shows that the financial market in reality works as an economic perpetuum mobile, where capital is created and destroyed without considerable changes in productivity. This again shows that thermodynamic analogies in economics are incomplete.

Our thermodynamic model for the financial market induces a rather unusual interpretation of the role of financial crises. In contrast to the common point of view, in our model financial crises play the crucial role in the functioning of the modern financial market. This is an important (concluding) stage of a financial cycle that is analogous to the stage of cooling in the ordinary thermodynamic cycles. A financial cycle could not be completed without such a stage as well as the ordinary thermodynamic (e.g., Carnot) cycle.

4 Maxwell Demon, Financial Version

Since in the situation we mentioned above profit is obtained due to using the difference in prices during the market cycle, which means that one sells and buys exactly when it will be profitable. Such an agent works as the Maxwell demon. The mentioned perpetuum mobile will be the perpetuum mobile of the second type (which violates the second law of thermodynamics). The standard objection against the Maxwell demon is that this demon should be in thermal equilibrium with the environment, and therefore it can not use thermal fluctuations of the environment in order to extract energy from the environment: it will itself have thermal fluctuations, and due to these fluctuations the demon will make mistakes, which will equilibrate with a possible gain of energy. But in an economy the Maxwell's demon looks quite possible. If the demon will have a temperature which is sufficiently lower than the temperature of the environment, then a number of mistakes which it makes will be low and the demon will be able to organize a flow of energy from the system. In an economy "temperature", which describes fluctuations, or noise in the system, corresponds to information about the market situation. It is quite possible, that some market agents will be more informed than the others. In this case these agents will be able to use their information to reduce fluctuations in their shares and act as Maxwell demons – to extract profit from the market.

We insist that this shows the way to extract a systematic profit which will not vanish after time averaging. From a physical point of view this corresponds to the fact that each new important information decreases the effective temperature of the market agent, and if one is able to perform the mentioned above thermodynamic-like cycle (or, at least, a considerable part of the cycle) before this new information will become common, this market agent will be able to work as a demon of Maxwell and extract systematic profit from the market. Moreover, since the financial market is a complicated nonlinear system, some market players will be able to organize the above mentioned thermodynamic-like cycles as a financial market. In this case they will have the needed information about these cycles and therefore will be able to extract profit from the cycles (if the price of organization of the cycle is less than the possible gain in the cycle. Or in other words, if the effectiveness of the financial heat machine is higher than the dissipation).

5 More Complicated Financial Thermodynamic-like Cycle

A primitive financial thermodynamic-like cycle presented in Section 2 can be modified to exhibit a more complicated behavior (which makes, in particular, less evident its presence at the financial market).

Consider following parameters: volumes of shares

$$V_1 < V_4 < V_2 < V_3 = V$$

and prices

$$P_4 < P_1 < P_2 < P_3 \, .$$

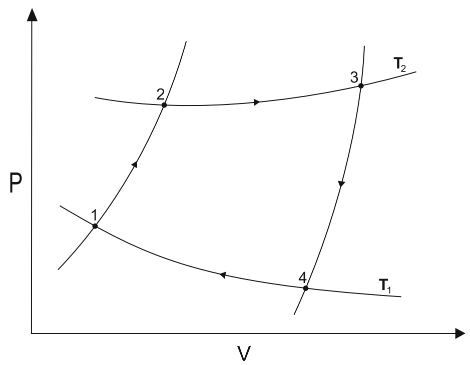


Fig. 2 (V,P)-diagram for a complex financial cycle

We assume that

$$V_2 - V_1 \ll V_3 - V_2$$

(the first difference is essentially less than the second), and

$$V_3 - V_4 \ll V_4 - V_1\,,$$

and that

$$P_3 - P_2 \ll P_2 - P_1, \quad P_1 - P_4 \ll P_3 - P_4\,.$$

We now consider a financial thermodynamic-like cycle having the same starting point as in Section 2. Thus there was totally issued $V = V_3$ shares, at the open market there were sold V_1 shares, G-people (designers) got $V - V_1$ shares. (The use of index 3 for V will become clear from coming considerations). At the beginning of the process of heating, the price is equal P_1.

Cycle:

1. Rise of quickly the price and slowly volume at the "open market": $1 = (V_1, P_1) \to 2 = (V_2, P_2)$. [Designers of this financial cycle start to sell their shares, the original volume V_1 at the open market increases, little bit.] The price curve $P_{12}(V)$ increases very quickly from P_1 to P_2. It is defined on rather small interval $[V_1, V_2]$.
2. Then very active sale of shares belonging to the financial group G at slowly increasing price $2 = (V_2, P_2) \to 3 = (V_3, P_3)$. Thus the price curve $P_{23}(V)$ increases slowly from P_2 to P_3 on sufficiently large interval $[V_2, V_3]$.
3. Then market crash $3 = (V_3, P_3) \to 4 = (V_4, P_4)$. Thus the price curve $P_{34}(V)$ decreases rapidly from P_3 to P_4. It is defined on relatively small interval, which is better to denote by $[V_3, V_4]$ (by taking into account cycle's direction). So, G-people even buy little bit at such a crashing market, but it is crucial that $V_3 - V_4 \ll V_4 - V_1$, i.e., this activity does not change essentially their profit.
4. Finally, G-people start to buy again shares at the very low price $P_4 : 4 = (V_4, P_4) \to 1 = (P_1, V_1)$. Here the price curve $P_{42}(V)$ increases slowly from P_4 to P_1. It is defined on relatively large interval, which is better to denote by $[V_4, V_1]$(by taking into account cycle's direction). Now they buy a lot! However, it is crucial the difference of the prices: P_4 is essentially less than P_2 and the price increases to P_1. [We remark, see Section 6, that real cycles are rarely based on shares of a single firm; typically a large part of the market is involved in such a cycle, G-people can choose what kind of shares to buy during this period, to prepare a new financial cycle.]

Curves $P_{ij}(V)$ form a closed graph in the (V, P)-plane, the financial cycle. At the first step of the cycle G-group gets the profit

$$A_1 = [\text{area under the graph of } P_{12}(V)]\,,$$

then

$$A_2 = [\text{area under the graph of } P_{23}(V)]\,.$$

After this G-people spent some money to buy shares for a new cycle:

$$A_3 = -[\text{area under the graph of } P_{34}(V)]\,,$$
$$A_4 = -[\text{area under the graph of } P_{41}(V)]\,.$$

Thus their profit in this more complicated financial cycle is given by a

$$A = [\text{area of the figure of the cycle}]\,.$$

Thus, as work in usual thermodynamics, the profit produced by this financial cycle is given by the area of the circuit 1234.

We will make a comment on the form of the graph. Unfortunately, we do not know the functional relation between the price P and the volume of shares (involved in a thermodynamic-like cycle) at the "open market", i.e., $P = f(V)$. In principle, such a relation can be founded on the basis of statistics, e.g., for the biotechnology or dot-com thermodynamic-like cycles. Nevertheless, we know that during the transition $1 \rightarrow 2$, heating the market, the price P increases essentially more rapidly than the volume V: during $2 \rightarrow 3$ (the constant temperature $T = T_2$) increase of V is essentially more rapid than the growth of P; during $3 \rightarrow 4$, P falls extremely rapidly and the volume decreases rather slowly (or even does not change at all as on the graph of Fig. 1; finally, during $4 \rightarrow 1$ transition ($T = T_1$), P increases very slowly with decreasing of V These general features of the process imply the form of the picture presented on Fig. 2.

Considered financial cycle is still an analog of an *ideal* thermodynamic cycle. In financial reality the situation is more complicated. At the real financial market designers of a heat machine need not try to come back to initial point 1, i.e., to prepare a new cycle by buying an essential volume of the same shares. They can escape expenses related to the part 3-4-1 of the cycle. First of all, as it was done in Section 2, they can stop any activity during crash. Thus they would not buy shares in the period 3-4. They can even escape costs of the period 4-1, by, e.g., reemission of shares.[8] Instead of reemission, which is profitable in the case of such "respectable corporations" as Eriksson, designers can just start creation of a new financial heat machine and to use profit from the completed cycle with say X-shares to start a new thermodynamic-like cycle with say Y-shares. In principle, designers need not be directly involved in the process of emission of shares. They can buy their portion of shares before to start creation of a cycle. In such a case their profits will be less. However, the cost of such an initial operation can be minimized by getting money from banks.

6 Complex Financial Heat Machines

The main deviation of the ideal financial thermodynamic-like model from the financial reality is consideration of shares of one fixed firm. A powerful financial

[8] I have a personal experience, very negative, with Eriksson's shares, reemission was the last step of Eriksson's cycle.

heat machine (intelligent designers are able to create such machines!) is based on shares of a large group of firms. Creation of large financial heat machines saves a lot of resources. To heat a group of shares is cheaper and sometimes easier than shares of a single firm. Among the most powerful financial heat machines we can mention e.g. Biotechnology-machine or dot.com-machine, see Malkiel 2003 for details. In the case of multi-shares financial heat machine designers buy shares (or participate in issues) of a large number of e.g. biotechnology-firms. Some of these firms have solid grounds, but some of them (if not majority) present really fake projects. Nevertheless, prices of all (at least of majority) biotechnology-shares go up as a result of "fruitful collaboration" with mass-media. After a cycle is done and profits are collected, designers need not to buy (even at very cheap prices) shares of biotechnology-firms. They can start to create e.g. the Information-technology-machine by using a part of profits from the previous heat machine. At the same time it may be profitable to buy shares of selected biotechnology-firms, which seem to do well in future. Of course, bankruptcy is the best way to decrease the volume of shares at the "open market". However, nowadays the use of this financial technology is restricted. Although bankruptcy provides quicker completion of a financial cycle, it is ineffective from the viewpoint of financial thermodynamics. Completion of a cycle through a huge wave of bankruptcies can be compared with functioning of the steam engine having a primitive exhaust system – directly to atmosphere.

The greatest financial heat machines have been created at American financial market. The state (by the way it often prosecutes creation of small heat machines) may actively participate in creation of huge heat machines, (cf. Malkiel, 2003).

The recent financial crises induced extended studies of the impact of interventions of respective Central Banks (CBs) to the financial market. The question of whether shifts in monetary policy affect the stock markets has been widely examined in both academic and policy circles, e.g., Petersen et al. (2009); Arouri et al. (2009). (The latter contains an extended review on this subject). It was shown that monetary policy innovations induce strong and significant responses by stock markets. This indicates that the investor community is keeping a close watch on CB intervention policies to make trading decisions. Relation between monetary stocks and stocks market is very complicated, see, e.g., for models. It depends on various parameters and it can vary from one market to another. For example, in Arouri et al. (2009) it was shown (by using nonlinear regression-type models) that reactions of the stock markets of USA and UK are similar, but different from France. For the UK, an interest rate increase immediately implies an increase of stock returns, but this will be absorbed after three days and become negative over four days. US stock market reactions after a shock affecting the US monetary market are similar to the UK. Phase durations and sizes are more significant in the USA. In Petersen et al. (2009), perturbations of the stocks market due to FED announcements were compared with (of course, of smaller scale) perturbations induced by crises. However, authors of all these papers were interested in short term effects of the monetary policy. Time scales considered in these works are too short to play a significant role in financial Carnot cycles. Now we perform simple behavioral analysis of FED-activity and the dynamics of Dow Jones Index in the period preceding the present crises as well during the crises, see http://www.federalreserve.gov/fomc/fundsrate.htm.

Between 03.01.01 and 25.06.03 the Federal Funds Rate was changed (by decreasing) 11 times. During this period Dow Jones Index was fluctuating demonstrating the tendency to go down. After this period it started to grow up. By our model it was the first stage – heating of the financial market.

Then FED started to increase the Federal Funds Rate. Nevertheless, as was already mentioned, Dow Jones Index grew up until the middle of 2008. Such a behavior contradicts the purely monetary model. However, it matches our socio-psychological model. At the hot market, each increasing of the Federal Funds Rate was considered (by agents of the market) as a confirmation that the hot stage was not yet over. Roughly speaking each of 17 announcements of FED during the period 30.01.04–29.06.06 was considered as a signal to buy financial assets. Thus the psychological consequence of FED's actions was opposite to its natural monetarist consequence.

Thus the period 03.01.01–29.06.06 and even until the middle of 2007 covered the stages (1,2,3) of the financial thermodynamic-like cycle at the market of the USA. FED's policy played an important role in forming of these stages of the present financial cycle. The constant temperature of the financial market was essentially based on FED announcements.[9]

Of course, the presented statistics can be interpreted in various ways. As was pointed by Petersen et al. (2009), and Arouri et al. (2009), FED's activity induces complex reactions of the financial market. Our main message is that one should sharply distinguish the purely monetarist model of these reactions from the behavioral-monetarist model. The first one implies that FED's activity in the period 30.01.04–29.06.06 did not induce any reaction from the financial market. In spite of permanent increase of the Federal Funds Rate, the stocks prices increased permanently. This was used as a critical argument against the regression model of Arouri et al. (2009), during the discussion on this paper at 68th Atlantic Economic Society Conference (Boston, October, 2009). However, as we shown, this FED's actions (which were practically meaningless form the viewpoint of the monetarist model, since they did not decrease stocks prices) might play an important informational role in the dynamics of expectations of agents operating at the market.

The role of state regulations in the financial cycles is a complicated problem. There is even the opinion that all crashes at the financial market of the USA were induced by financial regulation measures performed by the state, round table of 68th Atlantic Economic Society Conference (Boston, October, 2009).

[9] We remark that Dow Jones was very high even in fall of 2007 approaching 14.000 in October. Thus it cannot be used as a direct measure of the psychological state of the financial market. It is clear that at that time the process of cooling had been already started. The Federal Funds Rate approached the level at which it started to play its real monetarist role: 5.25% was high enough to start real cooling of the market. Psychological states of agents of the market were in the process of dynamical change, entropy of the market increased drastically approaching its maximal value, see Section 7.

Finally, we stress once again a role which finance rating corporations and experts play in financial cycles. Although they claim that they present independent opinion and refer to numerous scientific methodologies, in majority of cases they are deeply involved in boiling and cooling of the financial market (at the respective phases). Coming financial regulations should take such an activity into account. Rating structures should take a part of risks which clients meet because of really unfair recommendations of these structures.

7 Comparison of Thermodynamic (Carnot) and Financial Cycles

In spite of similarity between the thermodynamic Carnot cycle and the financial cycles which were considered in Sections. 2 and 5, it is important to remark that these cycles have different shapes. Deeper analysis of the differences of shapes is up to readers. In any event, the explored analogy between price and pressure is not complete. The financial heat machine does its cycle, but coupling between price and volume differs essentially from coupling between pressure and volume.

Nevertheless, we can try to continue to explore econophysical metaphor and consider curves $(2 \rightarrow 3)$ and $(4 \rightarrow 1)$ as corresponding to fixed financial temperatures. Here $T = T_1$ during $(4 \rightarrow 1)$ stage and $T = T_2$ during $(2 \rightarrow 3)$ stage, and $T_1 < T_2$. The financial temperature increases drastically during period $1 \rightarrow 2$ of development of the market, then it is constant $T = T_2$ (at least in the ideal model) during period $2 \rightarrow 3$ (here the market is very hot, sufficiently hot, there is no need to increase its temperature). Then the temperature goes down drastically during period $3 \rightarrow 4$, and finally, it is constant again, but it is very low $T = T_1$.

We now compare the physical (Carnot) and financial cycles of Sections. 2, 5 step by step. It is convenient to start with point 2. On the one hand, it is always done in thermodynamics. On the other hand, the part $1 \rightarrow 2$ is the main distinguishing part in comparison of physical and financial cycles. Therefore we prefer to discuss this part of the cycle later. The a) and b) sections refer to physics and finances, respectively.

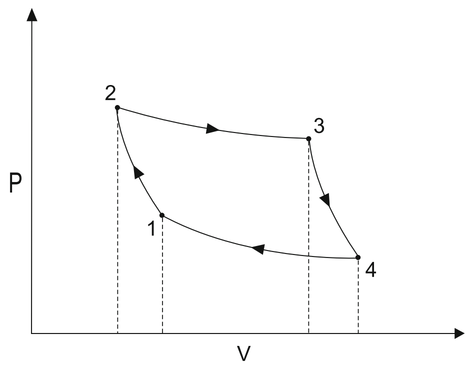


Fig. 3 (V, P)-diagram for thermodynamic Carnot cycle

$(2 \to 3)$:

a) Reversible isothermal expansion of the gas at the "hot temperature", $T = T_2$ (isothermal heat addition). During this step the expanding gas makes the piston work on the surroundings. The gas expansion is propelled by absorption of quantity Q_1 of heat from the high temperature reservoir.

b) The market is already hot enough, $T = T_2$. It is isothermal expansion of the volume of shares at the "open part of the market". The "heat" in the form of information from the mass media, experts, FED, CBs, and often even state authorities is needed to support the constant financial temperature and to do work for the G-group (financial Maxwell demons). Here the main work is done – the main profit is received:

$$A_{23} = \int_{V_2}^{V_3} P(V)\mathrm{d}V .$$

$(3 \to 4)$:

a) Isentropic (reversible adiabatic) expansion of the gas (isentropic work output). For this step the piston and cylinder are assumed to be thermally insulated, thus they neither gain or lose heat. The gas continues to expand, working on the surroundings. The gas expansion causes it to cool to the "cold" temperature, T_1.

b) The G-group starts cooling of the financial market. The financial temperature drops drastically approaching the isotherm $T = T_1$ at point 4. In contrast to, e.g., the Carnot cycle, no work on surrounding (the G-group in our case) done. The process is not adiabatic. The financial market is actively cooled.

$(4 \to 1)$:

a) Reversible isothermal compression of the gas at the "cold" temperature $T = T_1$ (isothermal heat rejection). Now the surroundings do work on the gas, causing quantity Q_2 of heat to flow out of the gas to the low temperature reservoir.

b) The market is cold $T = T_1$. The process is adiabatic. The price P increases (not essentially) as a consequence of decreasing (drastic) of the volume V. Now the surroundings (the G-group) do work on the financial market. Its work compress the volume of shares at the "open market."

$(1 \to 2)$:

a) Isentropic compression of the gas (isentropic work input). Once again the piston and cylinder are assumed to be thermally insulated. During this step, the surroundings do work on the gas, compressing it and causing the temperature to rise to T_2. At this point the gas is in the same state as at the start of Step 2.

b) This is the time to use a lot of fuel and to increase the financial temperature, from $T = T_1$ to $T = T_2$. During this step, the surroundings (designers of the cycle) do work on the financial market.

Thus, in contrast to the physical Carnot cycle, in the cycles considered in this paper the financial heat machine can not come back to the temperature $T = T_2$ without additional fuel. It is impossible to heat the financial market up to very high temperature only by decreasing the volume of shares at the "open market". The later strategy worked at ($4 \to 1$) stage, it provides some increase of the price, but it could not change market's temperature essentially.

The main difference between two cycles is exhibited during ($2 \to 3$)-stage. In both cycles V increases, but P (pressure) decreases in the physical world and P (price) increases in the financial world.

In the later case it is possible to increase both V and P. Moreover, P should go up, as at Fig. 2 or at least be approximately constant, as at Fig. 1. Any tendency of P to go down would induce decrease in sells of "hot financial assets."

It seems that the efficiency of the financial thermodynamic-like cycle is less than of the thermodynamic Carnot cycle. Of course, comparison of efficiencies of these two cycles is merely a metaphor. The main problem is to define properly a financial analog of heat Q, the basic quantity of thermodynamics. It would be natural to try to couple Q with money needed for "stimulation" of the mass media and financial experts as well as politicians (it can be shares, job appointment and so on). However, Q should also include activity of financial Maxwell demons who put tremendous efforts in each cycle. Thus Q-financial is not easy to measure. The situation is even more complicated. The Q which has been discussed is so to say G-related financial heat. The main quantity of total financial heat used during a cycle is produced by actors of the financial market, "simple people" buying and selling financial assets, traders and so on. Heat supplied to the financial market by the G-group plays an important role only during ($1 \to 2$)-transition. Functioning of the financial heat machine during ($2 \to 3$)-transition is covered merely by heat which self-induced, i.e., by actors of the market.

8 Entropy-temperature Diagram

We remind that in physical engineering the use of the entropy-temperature diagram provides a new insight to the Carnot cycle. It may be useful even in financial engineering. To analyze (S, T)-diagram (where S and T denotes financial entropy and

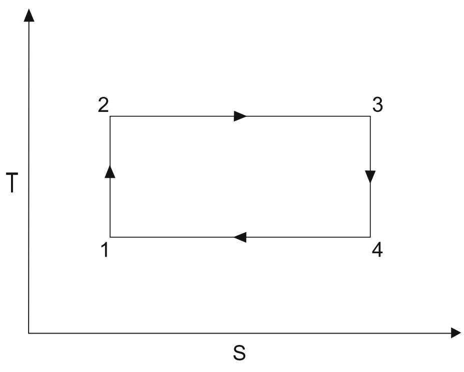


Fig. 4 (S, T)-diagram for the Carnot cycle and the financial cycles

temperature, respectively), it is useful to start with point 2. Corresponding (S, T)-dynamics for physical Carnot cycle can be found in thermodynamic literature. We remark that shapes of (S, T)-diagrams for the Carnot cycle for physical heat machines and the financial cycles considered in this paper coincide! Thus, in spite of essential difference in (V, P)-diagrams, these cycles coincide on the (S, T)-plane.[10]

However, the content of these diagrams is different (at least at some stages).

The basic variable under consideration is expectation of future dynamics of market. We quantify expectations, in the simplest case: negative, neutral, positive; in the general case of discrete quatification $x = x_1, \ldots, x_N$. Then we find probability distribution of the expectation-variable, $p(t, x_i), i = 1, \ldots, N$, and, finally, define entropy of this probability distribution:

$$S(t) = -\sum_{i=1}^{n} p(t, x_i) \log p(t, x_i) .$$

As always, the highest value of entropy corresponds to the uniform probability distribution, but concentration at one of points, e.g., positive expectation, minimize entropy.

$(2 \to 3)$: It is the period of isothermal expansion of the volume of shares at "open market". Entropy which was low at the beginning of this transition, $S(t_2) = S_1$ (see below $(1 \to 2)$-step for explanation) :increases during this stage. It approaches it maximal value, $S(t_3) = S_2$, just before point 3, the starting point of the financial crash.

$(3 \to 4)$: The crash occurs, market's temperature falls down, $T_2 \to T_1$. However, market's entropy does not change (at least essentially and at least in the ideal model), $S(t) = \text{const}$.

$(4 \to 1)$: This is the isothermal compression. Entropy decreases reflecting improvement of the positive attitude in expectations – under a clever supervision of mass-media, FED, CBs, state interventions. Of course, the process of improvement can be rather slow. In any event, entropy $S(t)$ decreases monotonicaly, approaching its minimal value at point 1, $S(t_1) = S_1$.

We now make a few additional comments on these steps in the financial cycle.

$(2 \to 3)$: There are two main sources of the increase of market's entropy. On the one hand, it increases because of internal evolution of expectations of individual traders. At point 2 common positive expectation of flowering of the financial market splits in a wide spectrum of expectations. On the other hand, the last stage of this transition is the stage of preparation of coming crash. CBs, FED, and mass-media start to send signals to prepare coming cooling of the market. It has not yet reacted properly to

[10] Surprisingly (V, P)-diagram for the ideal financial cycle of Section 2 coincides with its (S, T)-diagram (and consequently with (S, T)-diagram of the thermodynamic Carnot cycle). We cannot provide a reasonable explanation of this coincidence.

these signals. Therefore the prices of shares are still high, e.g., Dow Jones is still high (it may start to fluctuate little bit stronger). The main changes are done in expectations of agents of the financial market. Their psychological states are changed (in spite of stability of market's functioning). At point 2 their psychological states were sharply distributed around the state of strong optimism, say x_{optimism} – highly positive expectations of future development of the market. One of the simplest models can be based on Gaussian distribution d_{optimism} (which is continuous) with the mean value x_{optimism} and very small dispersion:

$$d_{\text{optimism}}(x) = \frac{1}{\sqrt{2\pi}\sigma_{\text{optimism}}} \exp\left\{-\frac{(x - x_{\text{optimism}})^2}{2\sigma^2_{\text{optimism}}}\right\},$$

where $\sigma_{\text{optimism}} \ll 1$.

Due to mentioned factors, dispersion of the Gaussian distribution increases; later even the form of distribution is changed, to camel-like distributions. Hence, entropy increases. Opinions and expectations become more diverse (they vary from expectation of a few more years of financial flowering to thoughts of a possible crash).

$(3 \to 4)$: During the crash the spread in expectations is still very large, since estimations of a possible character of the crises vary essentially depending on available information, measure of internal belief in mass-media, FED, CBs and state.

$(4 \to 1)$: The spread in expectation decreases slowly (again under careful treatment by mass-media, FED, CBs, and state). Hence, entropy decreases. At the same time market's temperature does not change (at least essentially). People arrive to point 1 with the feeling that the market will soon start its recovery. We remark that this happens only in their brains. The market considered as purely monetary machine does not exhibit signs of this preparation. Only the decrease of entropy shows that something happens (in spite of low value of e.g. Dow Jones). By using the Gaussian model we can say that the distribution of expectations approaches again d_{optimism}.

Remark. (Experimental research?) The previous analysis showed that market's entropy can be used as an additional important factor in estimation of market's state. Can entropy of expectations be found experimentally? Can it be monitored? It seems that it can be done. It needs sociological studies on expectations of agents of the market. Of course, the problem of quantification of expectation is quite complicated. However, it can be done. Monitoring market's entropy will provide a possibility to predict crashes and recoveries.

Conclusion

The use of thermodynamic-like models of the dynamics of the financial market demonstrated that financial heat machines can be designed in a similar way to the design of physical heat machines. The main difference is that functioning of financial heat machines is based on informational quantities, instead of physical quantities. Expectations of agents of the financial market play a crucial role. One can easily design financial heat machines which cycles are, in fact, cycles of expectations of future development of the market. Such machines provide a possibility of arbitrage. From the thermodynamic-like viewpoint complexity of the financial market includes complexity of complicated social relations which are used to design a financial heat machine. Market's strategies of decision-making induced by our model are strategies of decision making at the markets violating the efficient market hypothesis.

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A Physicist's Approach to Phase Controlling Chaotic Economic Models

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Abstract. We present a multi-frequency phase control able to preserve a periodic behaviour within a chaotic window as well as to re-excite chaotic behaviour when it is destroyed. The validity of this non feedback method has been shown in the cobweb model with adaptive price expectations as well as in the quadratic map near an interior crisis. A crucial role is played by the phase of the applied periodic perturbations

1 Introduction

Control of chaos represents one of the most interesting and stimulating ideas in the field of nonlinear dynamics. The original idea was to stabilize the dynamics over one of the different unstable periodic orbits visited during the chaotic motion with small periodic perturbations to the system. On the other side, one may wish to maintain chaos in regions where it is destroyed.

Here, we present a multi-frequency phase control method applied to a discrete economic model. The method is able to preserve periodic behaviour as well as to re-excite chaos previously destroyed as a consequence of tangent crises. The considered model is the " cobweb model " with adaptive price expectations [1–4] using parameter values where chaos and periodic windows appear.

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M. Faggini, C.P. Vinci (Eds.), *Decision Theory and Choices: a Complexity Approach*

2 The Cobweb Model with Adaptive Price Expectations

Let $q_n^d = D(p_n)$ and $q_n^s = S(\pi_n)$ denote the demand and supply of a given commodity. The cobweb model assumes for each n the condition$q_n^d = q_n^s$, representing a so called temporary equilibrium. Moreover the expected price π_n is assumed to be a weighted average of previous year's expected price π_n, and price p_n:

$$\pi_{n+1} = (1-w)\pi_n + wp_n \tag{1}$$

where $0 < w \leq 1$.

This model will be reduced to one recurrence relation containing a single variable. Solving Eq. 1 for p_n, substituting in the equation $D(p_n) = S(\pi_n)$, and subsequently solving for π_{n+1} yields

$$\pi_{n+1} = (1-w)\pi_n + wD^{-1}S(\pi_n) \tag{2}$$

Assuming a linear function for the demand D as $D(p) = a - bp$ and a nonlinear function for the supply function S as

$$S(\pi_n) = \arctan(\lambda((\pi_n - 1),$$

we obtain the following recurrence equation [4]:

$$\pi_{n+1} = (1-w)\pi_n + wa/b - w/b \arctan(\lambda(\pi_n - 1)) \tag{3}$$

The bifurcation diagram as a function of the parameter λ clearly shows chaotic behaviour and periodic windows originated by tangent bifurcations (fold bifurcations). Here we focus the attention on the periodic window emerging at $\lambda = 5.35$.

An expanded view of this region shows that as the control parameter increases , the stable fixed points undergo a period doubling sequence. A further increase of the control parameter the unstable fixed points come into contact with the chaotic bands and the chaotic attractor suddenly increases in size (see Fig. 2). The interior crisis, characterized by the merging of the five chaotic bands into a wider one is evident at the right end of window.

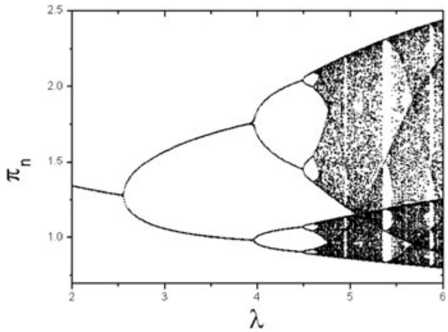

Fig. 1 Bifurcation diagram of the recursive relation in Eq. 1 $a = 1, b = 0.1$ and $w = 0.3$

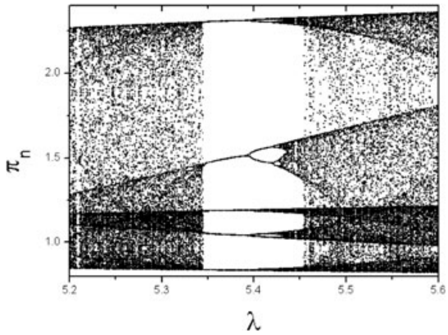

Fig. 2 Expansion of the bifurcation diagram shown in Fig. 1. A tangent bifurcation at $\lambda = 5.4$ gives rise to a stable and unstable (not shown) period five orbits. Successive period doubling bifurcations create five chaotic bands and finally their expansion in a broad chaotic attractor (interior crisis)

Control and sustaining chaos are obtained by slight modulation of the parameter λ. Such a perturbation consists of a sinusoidal signal of the following kind:

$$\lambda_n = \lambda\left(1 + \varepsilon \cos\left(2\pi r n + \phi\right)\right), \tag{4}$$

where ε is the strength of the periodic perturbation occurring at frequency ratio $r = 1/3$ and $1/5$ with respect to the intrinsic one. The advantage of this control approach, is that it is easy to implement in many time forced experiment, independently from time scales. Control of the period five solution, well inside the chaotic regions, is given through this method, for $\varepsilon =$ and $r = 1/5$ (see Fig. 3a).

The stability domain of the five-period orbit is greatly extended with respect to the unperturbed case except for isolated values of λ.

The effect of this harmonic perturbation is shown by plotting the bifurcation diagram as a function of the phase ϕ (see Fig. 3b).

The proposed method is also able to re-excite the chaotic attractor when it is destroyed by the appearance of a tangent bifurcation on the left edge of the periodic window.

By choosing $r = 1/3$, $\varepsilon = 0.9\%$ and $\phi = 1.5$ the chaotic behaviour is sustained within all the periodic window (see Fig. 4a and 4b).

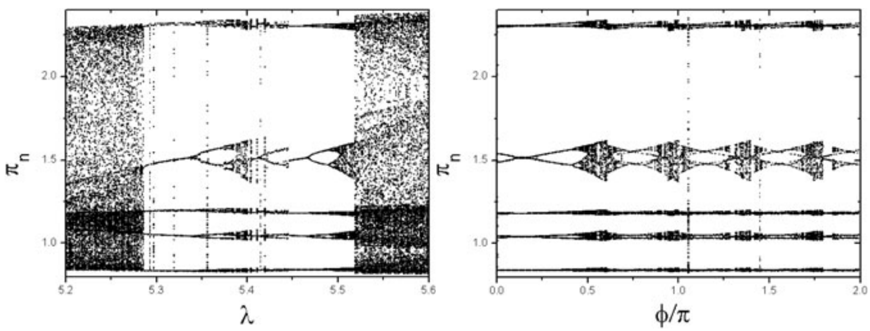

Fig. 3 a) Perturbed bifurcation diagram with $r = 1/5$, $\phi = \pi/5$ and $\varepsilon = 2.5\%$. b) Perturbed bifurcation diagram vs ϕ/π at $\lambda = 5.4$ when $r = 1/5$ and $\varepsilon = 2.5\%$

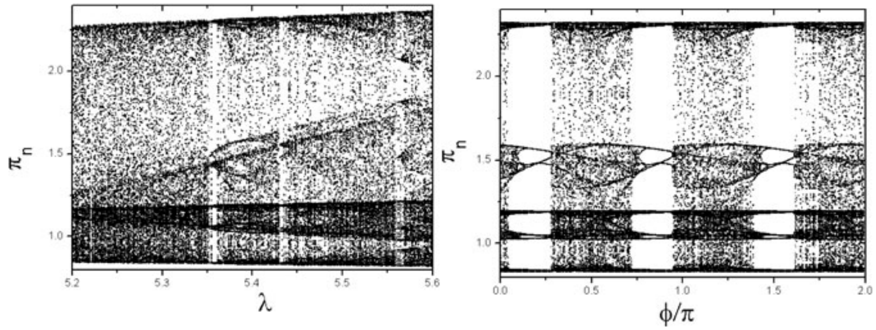

Fig. 4 a) Perturbed bifurcation diagram with $r = 1/3$, $\phi = \pi/2$ and $\varepsilon = 2.5\%$. b) Perturbed bifurcation diagram vs ϕ/π at $\lambda = 5.4$ when $r = 1/3$ and $\varepsilon = 0.9\%$

3 Phase Control in the Quadratic Map

In order to gain a deeper insight of the role of ϕ in nonlinear systems, we study phase control in a paradigmatic nonlinear map close to an interior crisis. We consider the unperturbed quadratic map given by

$$x_{n+1} = C - x_n^2 = F(C, x_n) \tag{5}$$

a paradigmatic system which is conjugate to the well known logistic map [5]. For this system, $C = C_s \approx 1.76$, a stable period three orbit is born. If we increase the parameter C , there is a period doubling bifurcation giving rise to three chaotic bands , but after a critical value, $C^* \approx 1.79$, the bands touch the unstable period that lies in its basin of attraction and they disappear (see Fig. 5a).

We are interested in the form of $F^3(C, x) \equiv F(C, F(C, F(C, x)))$ close to crisis. This form is depicted in Fig. 5b for $C \approx C^*$, together with the three points of the unstable period-3 orbit involved in the crisis x_i, which verify $F^3(C, x_i) = x_i$, where the subindex i will be a, b, or c. The key idea that we want to illustrate is that the existence of the three bands for $C \leq C^*$ and their disappearance at $C > C^*$ can be interpreted in terms of the evolution of the three unimodal maps that are present in $F^3(C, x_i)$. The conditions for these maps to be “ trapping” or not, can be given by the three following ratios:

$$R_i(C) = \frac{\left|F^3(C, x_{m,i}) - x_i\ \right|}{|x_i - x_{L,i}\ |}, i \in \{a, b, c\ \} \tag{6}$$

We can see that the bands will exist if $R_i \leq 1$ and they will disappear for $R_i > 1$.

Now we focus on the role of the phase ϕ when applying a harmonic perturbation to the quadratic map (Eq. 5). The perturbed map is

$$x_{n+1} = C_n - x_n^2 = F(C_n, x_n) \tag{7}$$

where $C_n = C(1 + \varepsilon \sin(2\pi r n + \varphi))$.

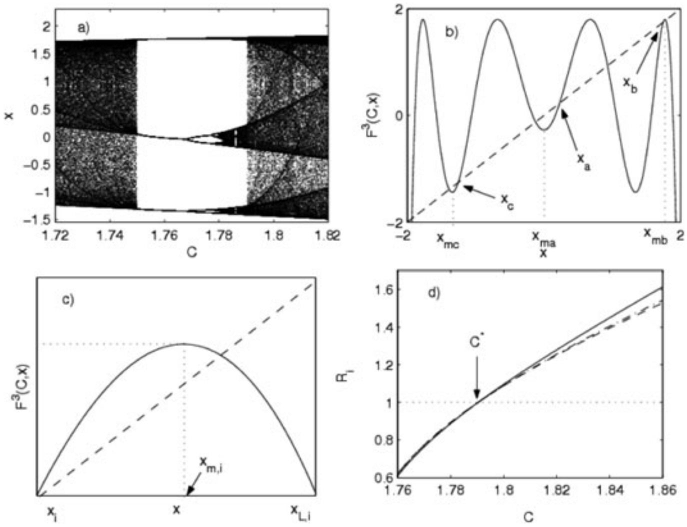

Fig. 5 a) Bifurcation diagram showing the interior crisis for the quadratic map. b) Plot of $F^3(C, x)$ as a function of x for $C \approx C^*$, where the unstable orbit x_a , x_b, x_b is marked. c) Scheme of each of the three copies of a unimodal map present in $F^3(C, x)$, responsible for the three bands observed in the bifurcation diagram. d) Plot of the numerical calculations of the three ratios $R'_i, i \in \{a, b, c\}$ which are smaller than one for $C < C^*$ and bigger than one for $C > C^*$

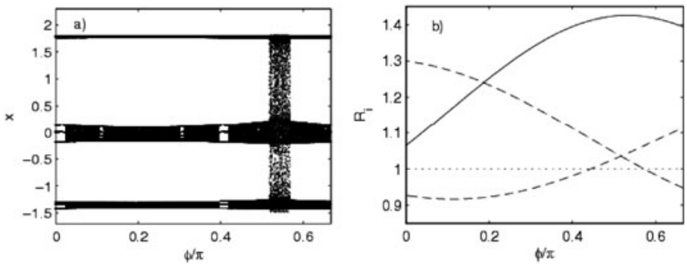

Fig. 6 Bifurcation diagram of $G(x)$ for $C = 1.8$, $\varepsilon = 0.005$ as a function of ϕ, showing that the value of the phase influences greatly in whether there is or not intermittency. b) Graphs of our approximation of the perturbed ratios R'_i as a function of ϕ: the ϕ interval where the intermittency is observed corresponds roughly R'_i to the interval for which $R'_i > 1$, as claimed

We will assume ε <<1. The case $r = 1$ is quite trivial , because, by varying the phase we are just moving in C in the interval $[C(1-\varepsilon), C(1+\varepsilon)]$.

Instead, in the $r = 1/3$ case the role of ϕ is far from being trivial. In this situation, the global dynamics will be governed by the autonomous map

$$x_{n+3} = F(C_2, F(C_1, F(C_0, x_n))) \equiv G(x_n). \tag{8}$$

The approximated values of the parameters R are now functions of C, ε and ϕ as

$$R_i' = R_i'(C, \varepsilon, \varphi).$$

The bifurcation diagram and a plot of our approximations of $R_i'(\varphi)$ are shown in Fig. 10 for $C = 1.8$ (the unperturbed system displays intermittency) and $\varepsilon = 0.005$.

Summarizing, we have seen that varying the phase ϕ modulates both the geometry of the system and the position of the periodic orbit responsible for the interior crisis.

4 Conclusions

The phase control of chaos is applied to the cobweb model with adaptive price expectation for parameter values displaying chaotic behaviour. The desired dynamics (control or sustaining chaos) is obtained by small periodic perturbations of the control parameter of the order of few percent with respect to the unperturbed value. The perturbation is applied to the "S-shaped " supply function. Both the amplitude and phase of the applied perturbation play a crucial role in selecting the goal dynamics. Such results are in agreement with those obtained in other physical systems [5–8].

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Part V
Related Issues

A Note on Complaints and Deprivation

Antonio Abatemarco

Abstract. In the existing literature a separating line is drawn between deprivation- and complaint-based inequality indices. In this paper we show that deprivation-based inequality orderings (eg. Gini index) can be replicated through the generalization of complaint-based inequality indices.

1 Introduction

In the existing literature several indices have been proposed in order to evaluate the allocation of income resources among members of the same society; different indices imply different orderings, and, as a matter of fact, economic policies might be strongly affected by the logical as well as analytical foundation behind one or the other index.

In the recent times two additional proposals have been considered: relative deprivation and complaints. Yitzhaki (1979) shows that the Gini index is a measure of deprivation in a society, by which inequality might be thought as the un-weighted aggregation of deprivation associated to each income position in the society. Instead, Cowell and Ebert (2004) formally elaborates Temkin (1986)'s idea by which inequality might be thought as some aggregation of individual complaints with respect to inequality coming from the sole disadvantaged individuals.

In this paper individual complaints with respect to inequality are defined taking the whole population as *comparative reference group*. This approach is found (i) to support "reference to the average" and (ii) to highlight the strong resemblance between complaints and net-deprivation. Under specific restrictions, the aggregation of individual complaints is found to replicate the orderings of some well-known inequality indices. In particular, a strong relationship is found between relative complaint orderings and the Gini's ones.

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M. Faggini, C.P. Vinci (Eds.), *Decision Theory and Choices: a Complexity Approach*

2 Basic Framework

Both complaints and relative deprivation involve income gaps among members of the same society. In addition, both concepts refer to a sort of *personalized* nature of inequality, that is, each income unit is expected to observe the allocation of income resources *in front of*, not *behind*, the veil of ignorance.[1] On the one hand, an individual is relatively deprived of income x when s/he does not have income x, s/he sees some other person or persons having income x, s/he wants income x, and s/he sees it as feasible that s/he should have income x (Runciman, 1966). On the other hand, an individual is expected to complain if s/he is disadvantaged within the income distribution. Also, his/her complaint is expected to be decreasing with respect to his/her own income.

As a matter of fact, an individual is deprived with respect to the sole income units richer than s/he is. In this sense the *comparative reference group* consists of some, not all, members of the same society. In addition, the *comparative reference group* is expected to vary depending on the reference individual. Complaints with respect to inequality, instead, refer to the way each member of the same society evaluates the allocation of the cake from the standpoint of his/her income position, so that each individual complaint implicitly involves comparisons with both poorer and richer income units, i.e. the whole population.

Given an increasingly ordered income vector $\bar{x} \in \Re^n_+$, if individual complaints are defined through the aggregation of pairwise income gaps where the *comparative reference group* consists of the whole population, then

$$c_k = \frac{1}{n}\sum_i (x_i - x_k) = \mu - x_k \tag{1}$$

where μ indicates mean-income. Individual complaints might be positive or negative, meaning that, within the society a clear separating line is drawn between disadvantaged and advantaged individuals. Also, each individual complaint is increasing with respect to incomes held by other members of the same society, that is, advantages and disadvantages are evidently the two sides of the same coin ($\sum_i c_i = 0$). Finally, (1) supports the first hypothesis proposed in Temkin (1986), by which complaints might be captured implementing "reference to the average".[2]

The most relevant implication of (1) is probably the intuitive relationship between individual complaints and relative deprivation. In order to highlight this aspect we briefly recall the definitions of relative deprivation and satisfaction.

Yitzhaki (1979) defines relative deprivation observed by income unit k as

$$d_k = \frac{1}{n}\sum_{i=k+1}^{n} (x_i - x_k) \tag{2}$$

[1] In this sense Runciman exploits relative deprivation in order to explain the political behavior of groups, not social justice.

[2] In Temkin (1986), this solution is defended as follows: "*In a world of* n *equally deserving people, the fairest distribution would be for each person to receive one nth of the total, since among equally deserving people, a fair share is an equal share.*"

As it has been clearly highlighted in Hey and Lambert (1980), this definition corresponds to average deprivation as obtained from pairwise income comparisons with respect to members of the k-th *comparative reference group*. Following the same approach, relative satisfaction might be defined as

$$s_k = \frac{1}{n}\sum_{i=1}^{k}(x_k - x_i) \tag{3}$$

by which, s is (i) increasing with respect to x_k, (ii) decreasing with respect to any increase of incomes lower than x_k, (iii) neutral with respect to rank-preserving rich-to-poor transfers below x_k and (iv) coherent with the null hypothesis, that is, if all income units poorer than x_k had been endowed with x_k, then relative satisfaction at x_k would have been null.[3]

Given the two definitions in (2) and (3), it must be the case that

$$c_k = d_k - s_k \tag{4}$$

In other words, individual complaints with respect to inequality (1) might be equally re-formulated in terms of net-deprivation. In particular, advantaged and disadvantaged individuals correspond respectively to net-deprived ($c > 0$) and net-satisfied ($c < 0$) individuals.

Once individual complaints have been identified, one may define the complaint-based inequality index as *some* aggregation of individual complaints. There are two immediate alternatives which might be considered. On the one hand, it might be said that advantaged individuals are *altruistic* in the sense that they too complain with respect to the current allocation of the cake.[4] As a result, in order to measure inequality, individual complaints would be

$$c_k = |d_k - s_k| \quad \forall\ k := 1, \ldots, n \tag{5}$$

On the other hand and in line with Temkin, since inequality might be thought as the aggregation of individual complaints from the sole disadvantaged individuals, the following notation might be preferred

$$c_k = \begin{cases} d_k - s_k & \textit{if } d_k > s_k, \\ 0 & \textit{otherwise.} \end{cases} \tag{6}$$

For our purposes we refer to the more general (5) since a return to Temkin's view is always feasible under neutrality of advantaged individuals.

The complaint-based inequality index might be defined as $C = \Phi(c_1, \ldots, c_n)$ where $\Phi : \Re^n \rightarrow \Re$ and $\Phi'(\cdot) > 0$. We assume that Φ is additive and separable. In particular, we impose symmetry since social complaints as well as inequality

[3] For additional considerations about (3) see Hey and Lambert (1980).

[4] In this scenario, very interesting insights might be gained from the recent literature on altruism and economics (Simon, 1993; Stark, 1999).

measures can be fairly required to satisfy invariance to permutations of the income vector.[5]

Recalling the well-known iso-elastic specification (Atkinson, 1970), the complaint-based inequality index would be

$$C = \frac{1}{n}\sum_{k=1}^{n}\left[\alpha + \frac{\beta}{1-\varepsilon}|\mu - x|^{1-\varepsilon}\right] \quad \varepsilon \in \Re,\ \varepsilon \neq 1 \tag{7}$$

where $\beta \geq 0$ can be assumed without loss of generality. In particular, we consider the more convenient and orderly-equivalent money-measure[6]

$$C = \left[\frac{1}{n}\sum_{k=1}^{n}|\mu - x_k|^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \quad \varepsilon \in \Re,\ \varepsilon \neq 1 \tag{8}$$

which, indeed, is just the compromise inequality index discussed in Blackorby and Donaldson (1978) and Ebert (1988). This index (8) is h.d.1, replication and translation invariant, that is, complaint orderings are both scale and translation invariant. Mean-crossing[7] rank-preserving rich-to-poor income transfers are always inequality reducing, that is, whatever $\Phi''(\cdot) \gtreqless 0$, any income transfer between the two reference groups in the society (from advantaged to disadvantaged income units) must necessarily reduce complaints in a society. More in general, rich-to-poor income transfers are always inequality reducing if and only if $\varepsilon < 0$, i.e. if the poorest income units are supposed to matter more than the richest ones at the margin, which is the standard ethical value judgement invoked within the welfaristic approach.

Obviously, if the sole disadvantaged individuals are involved in the construction of the complaint-based inequality index (Temkin's view), main implications of (8) apply to the sole income units such that $x_k < \mu$. In particular, rich-to-poor transfers among advantaged individuals would be inequality neutral.[8]

3 From Complaints to Inequality

In contrast with the impersonal approach to the measurement of inequality, both complaints and deprivation rely on the personal evaluation of individual complaints.

[5] Temkin (1986) briefly observes that in order to serve as inequality index it might be opportune to associate larger weights to the poorest income units. In this sense we think that a better framework might be obtained referring to the relative version of deprivation/satisfaction ((2)–(3)) as suggested in Chakravarty et al. (2003, 1985), by which individuals differently evaluates the same income gap depending on their own income level. Indeed, this approach would allow for both (i) larger weights for poorest income units and (ii) symmetric aggregation of individual complaints.

[6] The index is re-formulated in terms of its certainty equivalent.

[7] The poor income unit holds less than mean-income, not the rich one.

[8] This might be not the case whenever deprivation is not defined as the un-weighted aggregation of income gaps with respect to richer income units.

In the previous section we have mostly focused on individual complaints as defined in terms of *un-weighted* aggregation of income gaps. However, some additional notations might be introduced in the basic framework above. In particular, it might be observed that individuals do not usually attach the same weight to all pairwise income comparisons. Similarly, it might be said that each individual is more or less likely to get in touch with some members of the same society. In this sense, individual complaints might be defined as some *weighted* aggregation of income gaps. Then we briefly consider two reasonable hypotheses in order to generalize the basic framework discussed in the previous section.

The simplest generalization might be the following: individuals are differently inclined to relative deprivation and satisfaction, or, similarly, they have different probabilities to get in touch with poorer/richer income units. Then, focusing on the linear aggregation, individual complaints would be defined as

$$c_k = \tau_k d_k - \lambda_k s_k \tag{9}$$

with $\tau_k, \lambda_k \geq 0 \ \forall \ k$.

Given the social aggregator discussed in the previous section, $\Phi(\cdot)$, if advantaged individuals are supposed to be altruistic, then the complaint-based inequality index would be

$$C = \left[\frac{1}{n}\sum_{k=1}^{n} |\tau_k d_k - \lambda_k s_k|^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \qquad \varepsilon \in \Re, \ \varepsilon \neq 1 \tag{10}$$

Once again, the index is h.d.1, replication and translation invariant. Obviously, if $\lambda_k = 0 \ \forall \ k$ or $\tau_k = 0 \ \forall \ k$, the index is respectively deprivation- or satisfaction-based.

More difficulties arise when looking for a generalization of the effects of rank-preserving rich-to-poor transfers. This issue is not irrelevant, since, at the very least, an inequality index should be never increasing with respect to rich-to-poor income transfers. Depending on the parameters ε, τ and λ, the following cases might be of interest:

i $\tau_k = \lambda_k, \tau_k = \tau_i \ \forall \ i, k$.
Implicit orderings are equivalent to the ones obtained through compromise inequality indices (Blackorby and Donaldson, 1978; Ebert, 1988). Mean-crossing[9] rank-preserving rich-to-poor income transfers are C reducing. In addition, rich-to-poor transfers are generally C reducing if and only if $\varepsilon < 0$.

ii $\tau_k = \lambda_k \ \forall \ k$.
The complaint-based index is just equivalent to the one discussed in Cowell and Ebert (2004) under *reference to the average*. Mean-crossing rich-to-poor transfers are always C reducing. In general, rich-to-poor transfers are C reducing if i) $\varepsilon < 0$ and ii) $\tau_1 \geq \ldots \geq \tau_\mu \leq \ldots \leq \tau_n$.

[9] The poor income units holds less than mean-income and vice versa.

iii $\tau_k + \lambda_k = \text{const}$, $\tau_k \geq \lambda_k \; \forall \; x_k < \mu$, $\tau_k \leq \lambda_k \; \forall \; x_k > \mu$., $\tau_1 \geq \ldots \geq \tau_n$ e $\lambda_1 \leq \ldots \leq \lambda_n$.[10]
Mean-crossing rich-to-poor transfers are always C reducing. In general, rich-to-poor transfers are C reducing if $\varepsilon < 0$.

Out of these three particular cases, (10) may be well increasing with respect to rich-to-poor transfers.

A more general definition of individual complaints with respect to both (1) and (9) might be obtained whenever individuals are supposed to attach different weights to *each* pairwise income comparison, i.e.

$$c_k = \frac{1}{n} \sum_i (x_i - x_k)\theta_i \tag{11}$$

Within this framework it can be shown (appendix) that if (i) $\varepsilon = 0$, (ii) $\theta_i = \frac{i}{n}$ with $i := \{1, \ldots, n\}$ and (iii) advantaged individuals are supposed to be *egoistic* in the sense that they support the current (unequal) allocation of the cake (reduce the social complaint), then the following relation holds

$$G = 2\frac{C}{\mu} \tag{12}$$

where G is the Gini index and C/μ might be labelled as a relative measure of complaints in a society. Then, under these assumptions (linear aggregation, relative measurement and $\theta_i = \frac{i}{n}$) relative complaints' orderings would be just the same as Gini's inequality orderings.

4 Conclusive Remarks

In line with Berrebi and Silber (1985); Cowell and Ebert (2004), it has been observed that whenever complaints are exploited as logical foundation behind the measurement of inequality, several well-known inequality orderings might be replicated depending on the aggregation of pairwise income comparisons. In particular, the aggregation of income gaps allows for several insights in the field of perceived inequality, although a separating line should be drawn between personal and impersonal inequality measures.

[10] The meaning of (iii) might be clearer if τ and λ are taken as the probability for each income unit to get in touch (so compare their own income) with poorer or richer income units. In this context, the set of assumptions is equivalent to assuming that disadvantaged income units have larger as well as income-decreasing probabilities to get in touch with richer income units than they are, and vice versa. Then, under these hypotheses, rich-to-poor transfers reduce inequality. Conversely, rich-to-poor transfers might be not inequality reducing when poor income units have very low probabilities to get in touch with rich ones.

Appendix

Given an increasingly ordered income vector $\bar{x} \in \Re^n_+$ with $i := 1, \ldots, n$, the Gini's is

$$G = \frac{2}{n^2 \mu} \sum_{i=1}^{n} i (x_i - \mu)$$

From (11), if advantaged individuals are *egoist*, $\theta_i = \frac{i}{n}$ and $\varepsilon = 0$, then

$$c_k = \frac{1}{n} \sum_{i=1}^{n} (x_i - x_k) \theta_i$$

by which, since $(x_i - \mu) = \frac{1}{n} \sum_k (x_i - x_k)$, it must be the case that

$$C = \frac{1}{n^2} \sum_{i=1}^{n} i (x_i - \mu)$$

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Predictability of SOC Systems. Technological Extreme Events

Giovanna Bimonte

Abstract. The growth of societal networks of every kind (information, communication, transportation, etc.), the progressive interpenetration of natural and artificial systems, and the continually increasing complexity[1] of human organizations and institutions promises to magnify the impacts and to generate new types and combinations of technological extreme events. Given these observations, extreme events emerge as a powerful focus for organizing research activities that can advance scientific knowledge and benefit society.

The possibility to develop some new tools, as exploratory model, open new possibility research to forecast complex adaptive model and to predict Technological Extreme Events on SOC system.

Whereas complex adaptive systems and agent-based models of them originally seemed to pose a problem for policy analysis, they may also present an opportunity. The failure of computerized decision support systems to provide significant help for most problems is striking when contrasted with the impact of computer technology in other spheres. Looking back, we can now see that most policy problems involve complex and adaptive systems, and that for those problems the classical approaches of predictive modeling and optimization that have been used in decision support software are not appropriate. The next stage in the development of complexity science could well include a reformulation of decision theory and the emergence of the first really useful computer-assisted reasoning for policy analysis.

1 Introduction

Modeling is always an abstraction from the real world, and will not be perfectly completed as long as any kind of models. Especially in the social science, the mod-

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[1] I am very grateful to prof. Massimo Salzano, who has helped me with *complexity*.

els are exposed to the fact that individuals and society, which are targets to be modeled, make changes very often. It is caused by the abstraction at high level in social science. At the viewpoint of systemics, the rules on the layer would be changed by the emergence occurred from the lower layer. Thus, the model should be well flexible to change as the target changes. Consequently, the standard tools of analysis cannot be reliably used for complex systems.

With roots in many disciplines such as evolutionary biology, non-linear dynamical systems, and artificial intelligence, modern theories and models of *complex adaptive system* focus on the interplay between a system and its environment and the co-evolution of both the system and the environment. Subsequently, behavior in a complex adaptive system is induced not by a single entity but rather by the simultaneous and parallel actions of agents within the system itself. Thus, a complex adaptive system is self-organizing and undergoes "a process . . . , whereby new emergent structures, patterns, and properties arise without being externally imposed on the system" (Goldstein, in [19]). In other words, the behavior of a complex adaptive system is emergent. Emergence, then is "the arising of new, unexpected structures, patterns, properties, or processes in a self-organizing system. Emergent phenomena seem to have a life of their own with their own rules, laws and possibilities"(Goldstein, in [19]).

A complex adaptive system may be described as an aggregate of agents and connections, which suggests that some aspects of the complex adaptive system behavior can be deduced by various network theories, such as social network theory and graph theory (e.g. [6, 18]). For example, the level of connectivity in the network, in part, determines the complexity of the network. As connectivity increases, interrelationships, represented by chains of agents connected together in a contiguous fashion, also increase. At low and high levels of connectivity, the number of new inter-relationships increases slowly as connectivity is increased. At a critical value of connectivity, inter-relationships change dramatically ([9]). The number of interrelationships is significant because it indicates the potential for the network to engage in global communication from within; it also relates to its potential for chain reactions and effects at a distance.

The dimensionality of a complex adaptive system is defined as the degrees of freedom that individual agents within the system have to enact behavior in a somewhat autonomous fashion ([8]). Controls act as a form of negative feedback, effectively reducing dimensionality. On the one hand, controls, such as rules and regulations or institutional restrictions, ensure that an individual agent's behavior is greatly limited, thus, changing the complexity of aggregate behavior and helping the complex adaptive system to behave more predictably. On the other hand, when dimensionality is enlarged or when a higher degree of autonomy is given to agents to make decisions locally, outcomes are then allowed to emerge in a deviation-amplifying way or through positive feedback.

The environment exists external to the complex adaptive system and consists of agents and their interconnections that are not part of the given complex adaptive system. Operationally, the environment of a given complex adaptive system depends on the chosen scale of analysis. For example, if we identify a computer

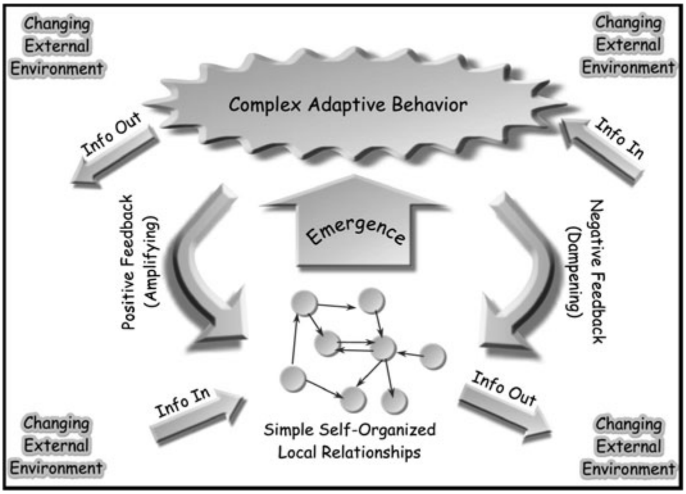

Fig. 1 Complex adaptive systems

network of a department in university as the complex adaptive system and users on the computer network as agents, then other users not on the computer network would form part of the environment. This environment would also consist of users in external computer network that they are connected to, as well as the socio-economic and cultural system in which the computer network is enveloped. Conversely, if we were to increase the level of scale and view collective bodies of teams as agents and the organizational boundaries as the boundaries of the complex adaptive system that would lead us to a slightly different identification of the environment.

In a complex adaptive system, it is often true that the only way to predict how the system will behave in the future is to wait literally for the future to unfold. Because behavior in a complex adaptive system stems from the complex interaction of many loosely coupled variables, the system behaves in a non-linear fashion. Non-linear relationships are relationships in which a change of given magnitude in the input to the system is not matched in a linear fashion to a corresponding change in output. In other words, if systems behave in non-linear ways, these behaviors naturally exhibit a non-linear response to changes. Therefore, in a non-linear system, large changes in input may lead to small changes in outcome, and small changes in input may lead to large changes in outcome ([10]). Essentially, a direct correlation between the size of cause and the size of the corresponding effect is not guaranteed. Thus, in practice, a complex system can be hypersensitive to small changes in its environment. A simple adaptive response, which usually leads to a simple corrective action, can lead to a catastrophic outcome (the so-called "butterfly effect"). Even if initial conditions and generative mechanisms are exactly specified (which they cannot be), prediction of the future often becomes fruitless as specification errors grow exponentially as one progresses into the future ([15]). The behavior of a complex system cannot be written down in closed form; it is not amenable to prediction via the formulation of a parametric model, such as a statistical forecasting model.

The inability to determine the future behavior of a complex system in an exact manner, however, does not imply that the future is random. Complex systems, in fact, exhibit patterns of behavior. It is true that small changes may lead to drastically different future paths; however, the same characteristic pattern of behavior emerges despite the change. One finds that systems will tend to be involved in certain prototypical ways and, thus, our predictive capacity, although limited to the exact prediction at a future point in time, can benefit from the knowledge of these patterns.

In this paper, we focus our attention on complex adaptive systems, as computer networks. In particular, we deal with viruses attack or bug on computer network as a *technological extreme events* that can happen in any nodes. The problem of attacks on complex networks has attracted a great deal of attention in recent years. The key factor prompting this research is the observation that in many growing networks some nodes evolve to become much more important than others. From a global perspective, the important nodes are those whose removal may either cause the network to fragment or severely limit the communication between the other nodes.

The paper is so organized: In Section 2, we take a look on Technological Extreme Events on Complex Adaptive Systems. We focus on the definitions of Extreme Events as emergent properties of interactions within or between complex systems and endogenous characteristics of it. Diverse extreme events may have similar longer-term consequences, as well. A computer virus and a flood may have nothing in common in terms of causes, but both might, for example, lead to the shutdown of a community's water treatment plant, with identical societal consequences and demands for response.

Reduced vulnerability can be achieved through predictions of trend-reversals or changes of regime are the focus of most complex adaptive systems efforts in essentially all domains of applications.

In Section 3, we deal with SOC systems. SOC is typically observed in slowly-driven non-equilibrium systems with extended degrees of freedom and a high level of nonlinearity. The main characteristic of nonlinear systems is that they are self-organizing. What is interesting is that these self-organizing systems have no focal points: they are mostly decentralized. Recent developments suggest that non-traditional approaches, can be useful for forecasting the evolution of this kind of complex systems. In particular, in Section 3.1. we consider social networks as SOC. They follow power laws and are characterized by many small events and few big events.

In Section 4, we consider predictability of SOC systems via Exploratory Models. The central idea behind Exploratory Modeling is to use simulations, statistical models of data, or other computer models to create a large ensembles of plausible images of the system of interest. Successful policies for complex adaptive systems will typically need to be adaptive themselves.

2 Extreme Events on Complex Adaptive Systems

Policy must act to reduce the human and economic effects of extreme events; research must provide knowledge and tools that can contribute to the effectiveness of policy. In the past, disasters as discrete phenomena that are external to the social or environmental systems upon which they impinge. According to this approach, extreme events and society are related to one another in a linear, cause-and-effect manner. The reality of rising extreme events impacts points to a different way to view extreme events: not as isolable phenomena, but as emergent properties of interactions within or between complex, dynamic systems. Extreme events are characterized and created by context. Extreme events was not inherent in any of the three ingredients of that tragedy: It emerged from their interaction; Every extreme events is in some way unique, which suggests that research strategies organized around particular types of extreme event might productively be reconceptualized in terms of a higher-level organizing principal. As extreme events linked to societal vulnerability via human decision making processes.

Our goal is to take advantage of emerging ideas from numerous disciplines to think about the linkages between scientific research and policy making relevant to hazard reduction, and in particular to define a perspective from which policy relevant research questions can be more readily recognized, and societally valuable knowledge can be more effectively created and used.

Definition 1. We define extreme events as occurrences that, relative to some class of related occurrences, is either notable, rare, unique, profound, or otherwise significant in terms of its impacts, effects, or outcomes.

The character of an extreme event is determined not simply by some set of innate attributes, but by the interaction of those attributes with the system that it is affecting.

Studying an extreme event independent of its context provides at best incomplete knowledge. In many complex adaptive systems, indeed, the extreme event does not exist independent of its context.

Extreme events reflect not just actions, but interactions, and the emergent properties of those interactions. Such complexity adds an even greater challenge to the scientific understanding and anticipation of extreme events and their consequences.

The threat of the *millennium bug*, for example, and the massive societal response to that threat, illustrated the comprehensive vulnerability to complex adaptive system. Technological extreme events created by society's growing dependence on complex information networks for managing its daily affairs. Yet the *millennium bug* problem was of a rare variety in that its cause and solution were rooted in a single factor the algorithm for embedding calendars into software that was both recognized and correctable in advance.

The growth of societal networks of every kind (information, communication, transportation, etc.), the progressive interpenetration of natural and artificial systems, and the continually increasing complexity of human organizations and institutions promises to magnify the impacts and generate new types and combinations of technological extreme events. Given these observations, extreme events emerge as a powerful focus for organizing research activities that can advance scientific knowledge and benefit society.

Diverse extreme events may have similar longer-term consequences, as well. A computer virus and a flood may have nothing in common in terms of causes, but both might, for example, lead to the shutdown of a community's water treatment plant, with identical societal consequences and demands for response.

Reduced vulnerability can be achieved through many avenues, including evolutionary processes (e.g., increased resistance to disease); preventing the extreme event (or modifying it to make it less extreme, e.g. Y2K reprogramming); reducing vulnerability before the event occurs (e.g., developed antiviruses); responding effectively to the extreme event after it occurs. Predictions of trend-reversals or changes of regime are the focus of most complex adaptive systems efforts in essentially all domains of applications. However, it is possibly the most difficult challenge.

Many kinds of approaches have been tentatively considered in the literature; SEVT (Statistical Extreme Value Theory), its POT variant (Peak Over Threshold) and the SOC approach, see [17]. The analysis of Extreme Events was traditionally founded on the assumption that this sort of event is exceptional. New trends of research argue that an extreme event is, in general, notable, rare, unique, profound and significant in terms of its impacts, effects, or outcomes. In fact, they claim that many such phenomena follow a power law for which extreme events are as normal as other kinds of events. This is the consequence of Self-Organizing Criticality (SOC). In the complex adaptive system of SOC, each extreme events is not characterized by preceding quantitative or statistical events that could allow its complex adaptive systemying, but by a structural modification of the laws underlining the system behaviour. Therefore, a complex adaptive system/decision theory based on "quantitative" past events is not suitable for a SOC systems. If the System is a SOC other approaches must be used. In [16] is showed that the results of pure statistical models could be erroneous when the system is a SOC.

3 Complex Adaptive Approaches in Self-Organised Criticality Systems

The main characteristic of nonlinear systems is that they are self-organizing. What is interesting is that these self-organizing systems have no focal points: they are mostly decentralized. For example, there is no central authority in the human brain, one that is in charge of all others. Accelerating up organic time via computer simulations of complexity theory, neural nets, and self-organizing evolutionary systems have been tried. However, most of the models were, understandably, realized using serial computer processing to simulate parallel behavior, as in the beehive.

Bak et al. [2] introduced first the concept of *self-organised criticality* (SOC), long-range spatio-temporal correlations that are signatures of deterministic chaos.

The power law distribution is:

$$N(s) \approx s^{-t}$$

where N is the number of observations at scale s and $t > 0$ is a parameter. The tenet of this approach is that "events can and do happen on all scales, with no different mechanism needed to explain the rare large events than that which explains the smaller, more common ones" ([3]). They can all be described as self-organizing systems that develop similar patterns over many scales. This is the consequence of the possible occurrence of coherent large-scale collective behaviors with a very rich structure, resulting from the repeated nonlinear interactions among its constituents. For this they "are not amenable to mathematical analytic descriptions and can be explored only by means of 'numerical experiments'", or simulations and often they are said to be computationally irreducible.

In physics, self-organized criticality (SOC) is a property of (classes of) dynamical systems which have a critical point as an attractor. Their macroscopic behaviour thus displays the spatial and/or temporal scale-invariance characteristic of the critical point of a phase transition, but without the need to tune control parameters to precise values.

SOC is typically observed in slowly-driven non-equilibrium systems with extended degrees of freedom and a high level of nonlinearity. A more simple way of saying it in relation to SOC is that small things happen very frequently and big things happen very rarely. And not just very rarely, but increasingly very, very rarely, the bigger they are. Self-organized criticality is one of a number of important discoveries made in statistical physics and related fields over the latter half of the 20th century, discoveries which relate particularly to the study of complexity in nature.

The recent excitement generated by scale-free networks has raised some interesting new questions for SOC-related research: a number of different SOC models have been shown to generate such networks as an emergent phenomenon, as opposed to the simpler models proposed by network researchers where the network tends to be assumed to exist independently of any physical space or dynamics.

SOC has been applied within a great number of disciplines, but it seems to remain a controversial subject for many scientists.

Self-Organized Criticality says basically that there are certain dynamic systems that have a critical point as an attractor. I.e. that they "by themselves" will move towards a critical state. There are certain characteristics to such a system, but more about that in a moment. One of the important discoveries about self-organizing critical systems is that they can be simulated quite well with cellular automata. Similar to how fractals can be generated with simple recursive formulas.

Recent developments suggest that non-traditional approaches, based on the concepts and methods of statistical and nonlinear physics, coupled with ideas and tools from computation intelligence, could provide novel methods in complexity to direct the numerical resolution of more realistic models and the identification of relevant signatures of impending catastrophes. Enriching the concept of self-organizing criticality, the predictability of crises would then rely on the fact that they are fundamentally outliers, e.g. viruses attack on computer networks are not scaled-up versions of a single computer corruption but the result of specific collective amplifying mechanisms.

3.1 Computer Networks as SOC Complex Systems

There are a few "laws" that typically are brought up when one discusses networks, particularly online social networks.

Networks such as the Internet have global reach with no central control. This lack of central control may make life harder for security personnel, but it does help survivability. In particular, the Internet has no single point of vulnerability that, if attacked, can bring the system down. The ability of the Internet to survive attack is strongly dependent on the manner in which its nodes are connected together.

In virus propagation, emergent properties again arise. When a virus is released, the manner in which it propagates over the Internet is difficult to predict. Simulation is a key contributor in helping understand or predict the Internet's response to virus attack.

The point at which information goes from a state of flowing to jammed is a critical point, a central topic in statistical physics and self-organizing criticality. The Internet is a complex adaptive system with many interactions. For example, more than 90% of its routers must be eliminated to make the structure disconnected. If less than ten percent of the most connected routers were removed, the infrastructure would decompose into many small, disconnected networks. Even if links were not removed, communication traffic jams could bring the Internet to a point of self-organized criticality affecting transmissions and packet flow.

Social networks seem to self-organize toward criticality. They follow power laws. There are many small events and few big events. All sorts of frequencies are mixed together. The network dynamically self-organizes itself into the most efficient state it could, without anybody being in charge.

Recent academic research in the field of complex systems suggests that the networks as well as computer networks or Internet self-organize under the competing influences of positive and negative feedback mechanisms. The equilibrium derive from the forces of negative feedback. When positive feedback forces dominate, deviations from equilibrium lead to crises. Such instabilities can be seen as intrinsic endogenous progenies of the dynamical organization of the system. Positive feed-

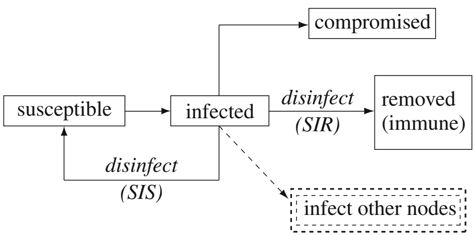

Fig. 2 Network survivability

backs lead to collective behaviour. This collective behaviour does not require the coordination of people to take the same action, but results from the convergence of selfish interests together with the impact of interactions between people through the complex network of their acquaintances. Complex system theory tells us that such collective behaviours may be very robust against external intervention, as long as the selfish individualistic nature of individual so called utility function dominates. The collective is robust because it derives from a bottom-up mechanism. Similar resilience is observed for instance in the Internet, due to its delocalised structure and self-organization. As a consequence, the origin of crashes is much more subtle than often thought, as it is constructed progressively by the market as a whole, as a selforganizing process. In this sense, the true cause of a crash could be termed a systemic instability.

4 Predictability and Exploratory Models

To make a model, the detail of the target model is understood during the process of making and simulating the behaviors on the computer, because of the complexity of the target. This incremental approach by trial and error is called constructive method in the science of complex systems. The researchers should build ad-hoc models for SOC systems. The researchers would choose some simulation models by exploring in all possible combinations of the alternative model components, before they make the simulation for the prediction and the analysis.

The central idea behind Exploratory Modeling [4, 5] is to use simulations, statistical models of data, or other computer models to create a large ensembles of plausible images of the system of interest. Search and visualization techniques can then be used to perform experiments drawn from this ensemble and to use the results of these experiments to provide information that is useful in distinguishing among alternative theories or strategies. Once it is realized that rigorous experimental science makes wide use of non-deductive arguments researchers are free to innovate a wide range of credible research strategies using computational experiments. Of particular significance is the ability, given a sufficiently powerful software environment, for users to interactively specify and observe the results of experiments, and to progressively define ever more general and abstract frameworks in which to describe such experiments. The researcher make assumptions which, together with any assumptions made in the construction of the model itself, constitutes a specification for a single computational experiment. Performing this experiment will reveal how the system being modeled would behave if all these assumptions were correct. By varying assumptions, the user can conduct a series of computational experiments. This series can be understood as an exploration through the *ensemble* of experiments that is defined by the set of all possible combinations of inputs. In such way, the output of the researcher is a patterns of outcome across the entire ensemble of possible experiments, and such patterns cannot be seen in the results of any single experiment.

Typical ad hoc approaches to research based on computational experimentation (exploratory modeling) confuses alterations which can be understood as exploring across the ensemble of models as originally defined and those which are best understood as revising the definition of the ensemble to be explored.

4.1 Ensemble

Best estimate models are constructed by using available knowledge about the system of interest. When such a model does not predict the behavior of a system, it is often true that there is additional information available about the system that was not used in constructing the model. Often, more information can be captured in an ensemble of alternative plausible models than can be captured by any individual model. Indeed, probability distributions are a representation of just such an ensemble. But, the restrictions that are imposed by the mathematical formalisms of probability theory can in computational modeling be avoided by a combination of explicit enumeration of finite lists of alternative options and inductive reasoning about the properties of infinite ensembles represented with generative techniques. qualitative and tacit knowledge held by humans and their organizations. Exploratory modeling allows such knowledge to emerge and be used throughout the course of an iterative analytic process. Consequently, it can provide a bridge for moving from deductive analysis of closed systems, to interactive analytic support for inductive reasoning about open systems where the contextual pragmatic knowledge possessed by users can be integrated with quantitative data residing in the computer.

Successful policies for complex adaptive systems will typically need to be adaptive themselves. Considering a unique policy based on the forecasts of single models results static and unfruitful. To test adaptive policies, a challenge set of possible future situations is needed, and the ensembles of alternative models being used for all of the previous techniques are perfect for this. Similarly, adaptive policies need to be evaluated on their robustness properties, not on their performance on any single case.

Further, the computer can be used to find important scenarios by searching through such ensembles, in particular to find cases that break a proposed policy. Such worst cases can stimulate users to modify the range of possible policies to allow for combinations that hedge against these possibilities. This strategy can allow users to iterate with the computer to gradually evolve policy schema that have particular policy instances with desirable properties.

The Exploratory Model is simple in its construction[2]; It is composed of the following steps:

1. Initial Specification Development, using information available.
2. System Construction or Modification.
3. System Test.

[2] See http://www.evolvinglogic.com/

4. System Implementation, after many interactions of the previous two steps produce good results, he system is implemented.

Exploratory modeling is the explicit representation of the set of plausible models, and the process of exploiting the information contained in such a set through a constellation of computational experiments. Exploratory modeling can be viewed as a means for inference from the constraint information that specifies this set or ensemble. Selecting a particular model out of an ensemble of plausible ones requires making suppositions about factors that are uncertain or unknown.

The problem of how to cleverly select the finite sample of models and cases to examine from the large or infinite set f possibilities is the central problem of any exploratory modeling methodology.

4.2 Exploratory Model for a Computer Network

A natural problem domain in which to study security externalities is computer viruses. To determine reasonable and cost-effective solutions to the virus problem, it is important to clarify the underlying goal. From the point of view of the individual user, the goal is obviously to avoid becoming victim of a computer virus. At the network level, however, it seems clear that complete prevention of all viruses is an unrealistic goal.

Though virus spreading through email is an old technique, it is still effective and is widely used by current viruses and worms. Sending viruses through email has some advantages that are attractive to virus writers:

- Sending viruses through email does not require any security holes in computer operating systems or software.
- Almost everyone who uses computers uses email service.
- A large number of users have little knowledge of email viruses and trust most email they receive, especially email from their friends.
- Email are private properties like post office letters. Thus correspondent laws or policies are required to permit checking email content for detecting viruses before end users receive email.

Considerable research has focused on detection and defense against email viruses. Anti-virus software companies continuously add new techniques into their products and provide email virus defense software[3].

To study this complex adaptive system formally, we must provide a way of precisely defining the component agents and their interactions. Exploratory modeling can be understood as search or sampling over a set of models that are plausible given a priori knowledge or that are otherwise of interest. This set may often be large or infinite in size. Consequently, the central challenge of exploratory modeling is the

[3] See, for example, the McAfee report *Mapping the Mal Web – 2008*.

design of search or sampling strategies that support valid conclusions or reliable insights based on a limited number of computational experiments.

The very huge amount of information available on technological extreme events on computer network, can be considered the basis for constructing exploratory model. In effect, data about a computer network serves to constrain the ensemble of plausible models, and our uncertainty corresponds to the size of that ensemble. When sufficient information exists to strongly constrain the ensemble of plausible models, reasoning can be supported by examining the properties of one exemplary case. As computer power becomes more abundant, we have a new possibility of reasoning about systems in which the ensemble of plausible models is less well constrained. Such reasoning must involve examining large numbers of examples drawn from this ensemble. The methodological question of how to select a limited sample from a large or infinite number of modeling experiments is only now beginning to be addressed.

Consider, now, the problem of a bug in a computer network. The behaviour of systems which contain a single bug, or a small number of them, is known to be governed by Poisson survival statistics [12]. For example, the probability pi that a particular defect remains undetected after t statistically random tests is given by

$$p_t = \exp(-E_t t) \ .$$

The quantity E_t is the 'power' of the defect, and depends on the proportion of the input space that it affects. Many systems that adapt to events in their environment can be described quantitatively in a similar way. The problem is that extensive empirical investigations have shown that in a large and complex adaptive system, the likelihood that the t-th test fails is not proportional to $\exp(-E_t t)$ but to k/t for some constant k[4] The same could be modeled for viruses spread on Internet.

Starting from this kind of model, and following steps of exploratory procedures, we can reach an ensemble of possible model, as possible states of world, and try to give some interpretation on the patterns of the computer network's or Internet vulnerability.

Note that even for models that are used to predict system behavior, there is always some residual difference between model prediction and measured outcomes in validating experiments. How small the residual difference may be and still have a "validated" model depends on the way the model is to be used. Further, for any model there are limitations to the range of uses for which the model is validated. Thus, no model is perfectly validated, and model validation must be assessed in the context of its intended use.

For exploratory modeling, validation centers on three aspects of the analysis: the specification of an ensemble of models that will be the basis for exploration, the strategy for sampling from this ensemble, and the logic used to connect experimental results to study conclusions. The specification of an ensemble of models of

[4] This was first measured by [1], who reviewed the bug history of IBM mainframe operating systems.

interest can be questioned in terms of the reasonability of the assumptions this specification embodies, with the biggest errors being those that unfairly exclude models from consideration. The strategy for sampling from this ensemble must clearly be rationalized in terms of the logic of the analysis if the results of computational experiments are to be useful. Thus, while individual models are subject to verification, it is studies that must be validated. To speak of the validation of models in an exploratory modeling context is incorrect and misleading. This situation is akin to that of actual experimental science, where the experimental equipment must be thoroughly checked to insure that experimental results are as measurement shows them to be (verification), but the validity of the research conclusions is based on a logic that connects them to the experiments that were actually conducted.

5 Concluding Remarks

In this paper, following [16], we have attempted to justify the use of modern modeling process, for the forecasting goal of the decision making. In our framework, complex adaptive systems characterized by self-organized criticality, statistical approaches for extreme events could fail prediction. Deep uncertainty, that characterized such complex systems, is the result of our limitation to use the representation formalism of statistical decision theory. The possibility to develop some new tools, as exploratory model, open new possibility research to forecast complex model. Whereas complex adaptive systems and agent-based models of them originally seemed to pose a problem for policy analysis, they may also present an opportunity. The failure of computerized decision support systems to provide significant help for most problems is striking when contrasted with the impact of computer technology in other spheres. Looking back, we can now see that most policy problems involve complex and adaptive systems, and that for those problems the classical approaches of predictive modeling and optimization that have been used in decision support software are not appropriate. The next stage in the development of complexity science could well include a reformulation of decision theory and the emergence of the first really useful computer-assisted reasoning for policy analysis.

Finally, we remark that, even if exploratory model are simple in its construction, they can be used in synergistic way with other classical statistical tools, as power-law distribution. As showed in [16] and [17], Soc systems forecasting needs new brand in research coordination between and within different disciplines and approaches.

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Ontology Based Risk Management

Giancarlo Nota, Rossella Aiello and Maria Pia Di Gregorio

Abstract. Risk management in several application domains is receiving increasing attention in the last years especially when the risk management must be pursued in a network made of interacting systems. The motivation is that although risk management models and techniques are mature enough to handle risk in the context of a single system, risk evaluation in the setting of a network of systems is much more difficult to model and manage. Because of the lack of awareness of risk, it is difficult to perceive risks propagation within the network of systems. On the other hand, the lack of shared goals and knowledge represents itself a risk, so that we need a good paradigm to organize and communicate information.

In this paper we first introduce a metamodel able to represent the fundamental structure from which distributed risk management models can be derived with respect to several application domains. This abstraction arises from an approach to risk management based on the definition of risk ontologies. A risk ontology is specialized to represent and share risk knowledge in a given application domain; changing the underlying ontology, the metamodel can be adapted to a new application domain so that the logic for risk management can be reused with a reasonable tailoring effort.

Two case studies are discussed in the paper as possible implementation of risk management systems based on the proposed metamodel.

1 Introduction

The term risk management is used in a wide variety of disciplines, and itself also combines concepts and techniques from a range of fields like statistics, economics, operations, research and decision theory [10] that can concern a single organization, a delimited geographical area or a distributed environment.

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Although there is not an universally accepted definition of risk, an abstract definition such as "the possibility of suffering loss" is general enough to be used in a wide variety of contexts. It implies the concept of uncertainty (an event may or may not happen) that combined with the concept of loss (an event has unwanted consequences or losses) gives rise to the fundamental definition of risk management known as *risk exposure* [5]:

Re = p(unwanted event)* loss(unwanted event)

Organizations can chose between many different methodologies and tools to be used at several abstraction levels for the management of risk exposure. Examples of widely used methodologies are: "Australian and New Zealand Standard for Risk Management" [4] that gives some guidelines for general risk management, the PMBOK [13] and PRMA that are guides explicitly set in a project management context while DOD-ISO/IEC standard emphasizes the important role of mitigation action.

Due to the great variety of application domains there exist also several definition of risk management. If we consider the field of Enterprise Risk Management, a suitable definition is provided by the business dictionary: "policies, procedures and practices involved in identification, analysis, assessment, control and avoidance, minimization, or elimination of unacceptable risks. A firm may use risk assumption, risk avoidance, risk retention, risk transfer, or any other strategy (or combination of strategies) in proper management of future events".

The literature concerning risk management in a single enterprise is mature enough. However, the research about risk management in a distributed context is at an early stage even if, in the last few years, the trend towards alliance partnership is constantly increasing.

Apart from aspects addressed by standard methodologies for risk management in a single enterprise, we must consider at least three further aspects about risk in a distributed scenario:

1. due to the shifty nature of risks in a cooperative alliance partnership, risk is difficult to perceive; for example, if an unwanted event arises in a given enterprise having a potentially negative impact on other participants, the risk exposure could remain latent until a loss become manifest;
2. when different organizations need to put in place a link between their information systems so they can exchange privileged information, it is necessary to manage the risks that such a link inevitably introduces [10];
3. each organization has its own manufacturing process, databases, setting and structure. In a distributed environment a certain number of organizations join together to perform a common project; the leader organization assigns activities to each partner together with responsibilities as well. Obviously, each organization manages a local risk management that is only a part of the risk management problem.

Therefore, it is necessary to establish common models, rules and knowledge, what information to exchange, which processes are relevant and what is the scope and nature of interaction between entities that participate in an interoperable risk management. *Interoperability* is defined as the ability of a set of communicating entities to exchange specified information and operate on that information according to a specified, agreed upon, operational semantics [11]. *Interoperable risk management* is defined in [11] as the subset of interoperable acquisition practices that enable acquisition, development, and operational organizations to identify, share, and mitigate risks that are inherent to a system of systems.

In an interoperable environment the organizations have to be able to share and quickly communicate about risk, to establish the nature of agreements among the entities, to detect what risk management information is needed to share and what operations or behaviours are related to risk management and must be performed in order to avoid or mitigate risk.

In this paper, we first propose a possible abstraction that will give us the possibility to reuse a distributed risk management metamodel in several application domains; then we show two case studies as possible metamodel applications.

The metamodel proposed in this paper arises from the fusion of two models: one is based on a risk ontology whose aim to capture the fundamental concepts of risk management together with a formal specification of rules to qualify operational aspects of risk management; the other is oriented to the assignment of activities and of the corresponding risk management responsibilities in a distributed environment.

By adopting a formal ontology and specifying the behaviour of a risk management system by means of transition rules, we obtain three goals: a deeper understanding of distributed risk, the check of the domain and knowledge sharing. This approach can be exploited in various contexts: adopting a tailoring phase that allows to qualify specialized ontologies every time the context changes, the metamodel can be reused. In the following we discuss two case studies concerning the monitoring of environmental risk and the management of risks about distributed software projects. We will show how the metamodel can be applied in both cases by conveniently replacing the ontology.

After some preliminary concepts discussed in Section 2, the metamodel for distributed risk management is introduced in Section 3 and two case studies, specialized as particular instances of the general metamodel, are discussed. The sections dedicated to case studies show how the general metamodel can be used as a guideline to specify a reactive risk management system in the given application context. The analogies between risk management systems represented in the metamodel can thus be reused allowing the implementation of a new risk management system with a reasonable tailoring effort.

2 Preliminary Definitions

The metamodel presented in this work is built on few fundamental concepts and on the well known risk management methodology, introduced by the Software Engineering Institute, to handle risks in software engineering projects. We first resume the main characteristics of the SEI paradigm [9], then we discuss the structure of a risk matrix as well as the structure of a risk ontology. These fundamental concepts will be used as building blocks for the definition of the metamodel for the distributed risk management presented in the next section.

Risk is a continuous process that takes place throughout the life of a project. Risks have to be continuously identified; therefore, a constant vigilance on the entire life cycle of project is necessary [12, 13].

The SEI paradigm shown in Fig. 1, illustrates a set of functions that are identified as continuous activities throughout the life cycle of a software project:

- **Identify:** consider risks before they become problems
- **Analyze:** convert data into decision-making information
- **Plan:** decide what should be done about a risk or a set of related risks.
- **Track:** monitor risk indicators, acquire risk status and record it.
- **Control:** decide for the best reaction when the risk probability increases or when unwanted events happen.

The **Communicate** activity is a cross one in the sense that data or information handled by a certain activity can be communicated to the involved stakeholders with the purpose of maintaining risk and loss under control.

Continuous Risk Management is a software engineering practice with processes, methods, and tools for managing risks in a project. It provides a disciplined environment for proactive decision-making to:

a) assess continuously what can go wrong (risks);
b) determine what risks are important to deal with;
c) implement strategies to deal with those risk.

In order to evaluate risk priority and to put risk under control, usually risk managers use the model known as risk matrix, where cells representing fuzzy risk exposure values are grouped in a given number of classes. In this paper we will refer to the classes shown in Fig. 2: **L** means low, negligible risk, **M** indicates a moderate risk, **H** a risk with high impact and probably high loss, and **E** represents the class of intolerable, extreme risk with very likely loss. Obviously, when the impact or likelihood grows, or both, the risk consequently grows; so a risk can modify its position from a lower category to an upper category. For each category of risk exposure, different actions have to be taken: values **E** and **H** involve a necessary attention in

Fig. 1 The SEI risk management paradigm

Fuzzy value	Range
Rare	0÷0.05
Unlikely	0.05÷0.2
Moderate	0.2÷0.4
High	0.4÷0.65
Very high	0.65÷1

(a) Likelihood: fuzzy values

CONSEQUENCE \ LIKELIHOOD	Rare	Unlikely	Moderate	High	Very likely
Very high	H	E	E	E	E
High	H	H	E	E	E
Medium	M	M	H	H	E
Low	L	L	M	H	H
Very low	L	L	L	M	H

(b) The Risk Matrix

Fig. 2 Building the risk matrix

priority management and a registration in the mitigation plan; a value **M** requires to be careful during the whole project management; a value **L** falls within ordinary management.

One of the most important problems to solve in a network of cooperating entities is the possibility of misunderstanding between entities about definitions and actions to take. The sharing of standard knowledge by all the communicating entities is an useful tool that allows to alleviate the misunderstanding problem; as shown in the next section, we propose to use of a risk ontology to share risk management knowledge within a network of cooperating entities (e.g. network enterprises or network sensors).

An ontology is a formal representation of a set of concepts within a domain and the relationships between the concepts. It provides a shared vocabulary to model a particular domain by means of a set of entities, properties and classes; in this way the same interpretation of the facts among people and organizations belonging to a particular context is guaranteed even if there are several distributed sources of information. Moreover, through a formal ontology, a deeper understanding of distributed risk is possible to increase the awareness of the systematic interdependencies; therefore, a distributed risk management metamodel based on risk ontologies is proposed in this paper.

In the following we will show the validity of the metamodel in two cases studies showing that, changing ontology and the logic transition rules, the metamodel can be adapted to different scenarios.

3 A Metamodel for Distributed Risk Management

A sensor network can be used in a wide scenario of applications ranging from environmental monitoring, climate control, structural monitoring, medical diagnostics, disaster management or emergency response [7]. It consists of spatially distributed autonomous devices using sensors to cooperatively monitor physical or environmental conditions, such a temperature, sound, vibrations etc. at a different locations.

Indeed, adopting the point of view usually considered in cybernetics, the concept of sensor can be used in a broader sense indicating an individual unit that inputs data [15], either an electronic device or an human being, that collects data from the external environment.

For their own nature, these monitoring applications are devoted to manage intrinsic risks that might arise in the systems in which the sensor network operates. Therefore, an effective and efficient risk modeling phase assumes great relevance for the prevention and the mitigation of undesirable and/or dangerous events.

We claim that many distributed systems have strong analogies from the risk management perspectives and that a suitable metamodel can catch the common properties and behaviour of several risk management systems. These analogies can be represented in an abstract metamodel.

When the metamodel will be used to approach the risk management in a given application domain, a tailoring phase will be necessary to handle the peculiar aspects of the system to put under control.

This point is further discussed in the next sections where two possible instances of a risk management system, operating in different context, are derived from the general metamodel through the application of a tailoring phase.

Figure 3 shows the metamodel of a distributed sensor network for the representation of the risk management problem in a distributed setting. Each Local Monitoring node $LM_1, LM_2, \ldots, LM_j$ of the network is responsible for the risk monitoring of a specific locations and is connected to the others by a communication infrastructure. Considering the generic Local Monitoring node LM_i, sensors $S_1^i, \ldots, S_n^i$ captures data from the environment and send them to the associated node for the data analysis and risk evaluation.

At a given time instant, a node LM_i manages one or more risk lifecycles, according to the SEI Paradigm, that are associated to specific risks to be monitored. A higher level Global Monitoring node GM, is responsible for the global monitoring of the network; apart from the typical functions assigned to local monitoring nodes, it executes further functions:

- ontology definition
  - identify, code and share risk classes;
  - analyzes risks and specify the behavioural logic for risk management;
- plans and configures the network of LM_i
- assigns responsibilities to LM_i in terms of activities and risks to manage
- capture alarms that involve two or more local monitoring nodes.

To specify the behaviour of a risk management system charged to manage events with a possible negative impact on the correct execution of activities, we will use a rule based logic language called RSF [1, 8]. With this language a reactive system can be defined in terms of *event-condition-transition rules* able to specify systems

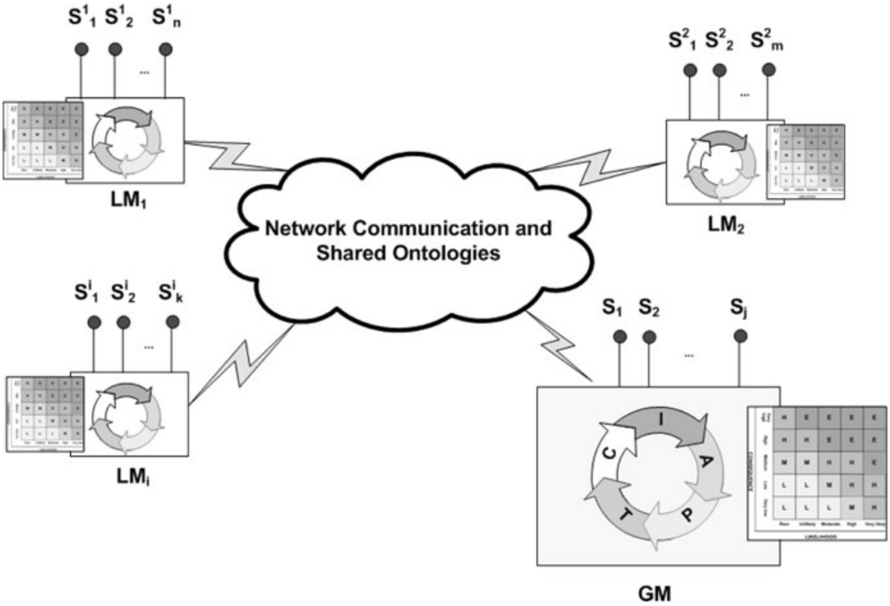


Fig. 3 The sensor network metamodel distributed risk management

requirements subjected to temporal constraints. The risk specification as defined in RSF can drive the design and the implementation of distributed system for the treatment of risk. The system will be able to react within given times to mitigate risks or to adopt emergency plans before that manifested risks can produce damages reputed not tolerable.

Below, we show some abstract metarules that specify some common behavior of many risk management systems. The availability of a set of metarules that qualify in abstract the reactive behavior of a risk management can be taken as a reference to implement a risk management system in different application domains.

At each time, the risk management system records a state concerning various kinds of data about risks. When an unwished event with a negative impact on the system under control is recognized, the risk management system adjusts its state to record the occurrence of a risk event and, eventually, it reacts activating a mitigation action. The metarules shown below are defined for a general entity called "node" but they could be tailored to monitor, for example, a project activity, a subsystem, a geographical area etc.

The first one states that when in the system state is present a variable that stores the risk exposure value of a given kind of risk, according to the matrix, and in the system, one or more new risk events appear, the transition rule activates itself. First, it erases from the state the risk exposure and the list of risk events and produces a new risk exposure value that depends from the old risk exposure and the list of risk events.

MR1: increasing the risk exposure value
from riskExposure, riskList
cons riskExposure, riskList
produce riskExposure **with** newRiskExposure(riskExposure, riskList)

In many cases, when a certain risk value increases, becoming high or extreme, one or more alerts have to be send. Sometimes, when the risk monitoring involves a single node, its control must be locally managed; in other cases, when the risk has an impact on two or more nodes, it is necessary than a higher level role must be alerted in order to proceed with a mitigation action. *MR2a* and *MR2b* handle with these eventualities.

MR2a: local alert
from riskExposure **with** riskExposure= "high" or "extreme"
produce alert(responsible role)

MR2b: global alert
from riskExposure **with** riskExposure= "high" or "extreme"
produce alert(GM)| alert(other global responsible roles)

To acquire more knowledge about risks, a $node_A$ can ask some information to another neighbouring $node_B$. As soon as the request is sent, the node awaits a response for a certain time ΔT.

MR3: exchanging information between nodes
from needinfo(riskExposure)
produce requestInfo(riskExposure, $node_B$), waitResponse($node_B$, ΔT)

If no response arrives before the time ΔT expires, an escalation action will be executed [2].

MR4: escalation on incomplete information
from waitResponse(node, T)
    not-occur receive(node, T_1) **with** $T_1 \leq T$
produce alert(GM)| alert(other responsible roles)

As the interoperable risk management is one of the most important aspect that a support system has to assure, the metamodel can address this aspect by means of appropriate metarules. Assuming that there is a dependency relationship between $node_A$ and $node_B$, the example below shows a metarule for the risk propagation from a $node_A$ to another $node_B$. *MR5* is attached to $node_A$ and defines part of its behaviour with regard to risk propagation. In its turn, the $node_B$ handles these events applying the metarule *MR1*.

MR5: risk propagation
from riskExposure, riskList
produce riskList($node_B$)

In the following sections we will show the two steps tailoring of this general model to the case studies. First we design the class diagram of the shared ontology; then the metarules are instantiated to the chosen application domain.

4 Case Study 1: Risk Management in a Distributed Software Projects

The case study presented in this section is a particular instance of the general model shown in the previous section applied to risk management during the progress of a software project pursued by a Virtual Enterprise.

A Virtual Enterprise (VE) is a temporary alliance of heterogeneous enterprises that want to adapt their business to the market changes; they join their specific competences to manage a common project and address a specific business target [14].

Due to the distributed nature of the examined context, a local risk management must be executed in each participating enterprise; a global activity of risk management is also necessary to control and mitigate risks that might negatively affect the reaching of common goals.

According to the metamodel discussed in the previous section, each enterprise can be considered as a Local Monitoring node LM_i capable of evaluating and handling the different risk typologies under observation.

The Leader Organization has the responsibility of the whole project coordination and control. In terms of the metamodel, it assumes the role of Global Monitoring node GM and assigns, to each partner, a set of activities that must be monitored from the point of view of risk exposure.This scenario is represented in Fig. 4.

In the distributed network for the monitoring of software risks, the role of sensors is played by human beings that collect risk data gathered from their own organization; risk data gathered by a single organization are first analyzed; if the risk has an impact local to the single enterprise, then the risk data is managed by local business rule; otherwise, the risk has a global impact and risk data are sent to the project manager who can pursue the risk resolutions according to a global risk management plan.

In order to show how the metamodel introduced in the previous section can be specialized to the problem of risk management in a VE, we first discuss the ontology shown in Fig. 5 together with the RSF rules that an ontology based support system can use to manage risk events. The approach chosen for the management of risks in a VE is focused on the project activities. To each activity is associated one or more risk matrix that provides data about the current value of the risk exposure. Such a value can change as an unwanted event becomes manifest with an impact on the considered activity. For the purposes of this work, we will consider three kinds of risk matrix: schedule, artifact quality and cost of a given activity.

The formal description shown in Fig. 5 reflects exactly our idea about the central role of activities and risks to put under control risk and risk exposure.

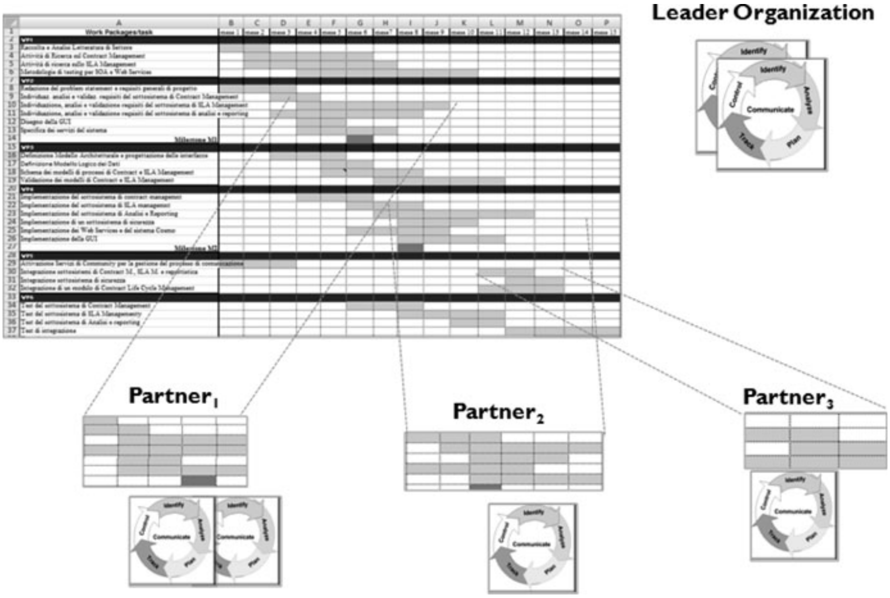
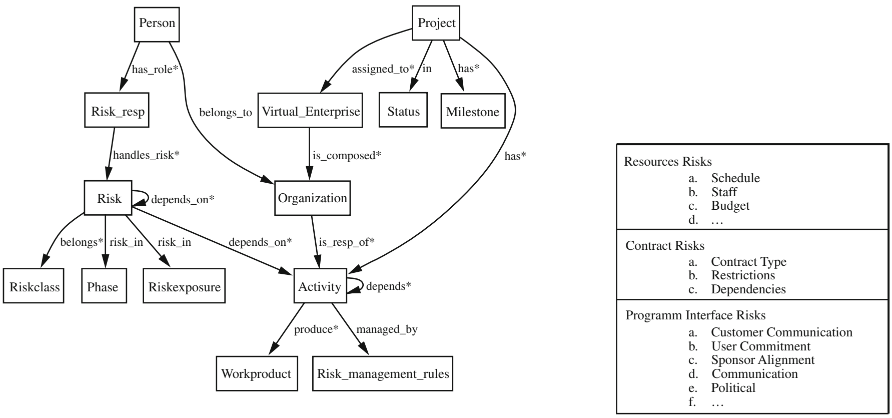


Fig. 4 Assigning activity and risk management responsibilities for distributed software projects



(a) Class diagram (b) Risk classes

Fig. 5 A risk ontology for the project management in a VE

The class *Activity* represents the activities planned to carry out a *Project* that, at a given time, can be only in one status (`new`, `started`, etc.); each activity can be assigned to an *Organization* belonging to a *Virtual Enterprise*. Figure 5a puts into evidence the dependences between activities, risks and between activities and risks. Indeed, it might happen that an activity can depend by one or more activities as well as a risk can depend by one or more risks. The risk taxonomy is defined according to SEI classification for software project (*Riskclass*); the Fig. 5b shows an excerpt of frequently used classification about programmatic risk. The fuzzy classification provides the qualitative determination of risk exposure (class: *Riskexposure* that can assume values `low`, `medium`, `high` or `extreme`); the class *phase* is referred to the SEI functions (`identify, analyze, plan, track, control, communicate`). Furthermore, risk responsibilities are established in order to assign the risk management to a person with specific skill (*Risk_resp*).

The first example shows an RSF rule that deals with the monitoring of a schedule risk related to an `activityA` under the responsibility of a participating organization to a VE. The rule is written in RSF language. Let us assume that the system state S_0 of the distributed system has the following configuration:

{ S_0 = < {<riskExposure, [scheduleRisk, activity$_A$, low],3>
<riskEvent,[serverCrash, scheduleRisk, activity$_A$],8> . . .], 8>

It represents the presence of a risk exposure value "low" for a schedule risk of the `activityA` at time 3 and the happening of a risk event "serverCrash" with a negative impact on the schedule of `activityA` starting from time 8.

In the example, the risk is assumed to be local to an organization and does not propagate to other organizations. The rule *R1* states that when in the system state the `riskExposure` for the `activityA` has a certain `currentValue` at time T_1, if a risk event happens at time T_2 (as described in the **from** clause in the rule), the rule is applied; first, data about current risk exposure and risk event are deleted from the state (**cons** clause in the rule), then a new state in the system is obtained immediately (after a null delay) producing a `newValue` of `riskExposure` for `activityA`.

R1) Local Risk Monitoring
from <riskExposure,[scheduleRisk, activityA, currentValue],T$_1$ >
<riskEvent,[Risk, scheduleRisk, activityA],T$_2$ >
cons riskExposure,riskEvent
produce <riskExposure, [scheduleRisk, activityA, NewValue],0>
with newRiskExposure(currentValue, Risk, NewValue)

Then rule *R1* is applicable in S_0 because the *from* clause matches some events structure present in the state; rule variables will assume the following value:

```
Value=low
Risk=servercrash
T1=3
T2=8
```

After the rule application, the risk exposure connected to `activity`$_A$ is evaluated again applying the function `newRiskExposure` that, starting from the current risk exposure and from the type of the risk event (`low` and `servercrash` respectively in the example), computes the `newValue` of the risk exposure for the `activity`$_A$. The current risk exposure values changes, for example, from a `low` value to a `medium` value according to risk matrix reported in the Section 2. This change is registered in the new state S_1 derived from the rule application S_1 = <{. . . <riskExposure,[scheduleRisk, activity$_A$, medium],8>}.

The second example considers how risk dependencies can be handled using RSF rules. In Fig. 6 a possible assignment of activities to organizations participating to a VE is shown with activity dependencies that cross the organization boundaries.

R2) Risk dependencies handling
from <riskExposure,[scheduleRisk, activity$_A$, Value],T$_1$ >
<riskEvent,[Risk, scheduleRisk, activity$_A$],T$_2$ >
cons riskExposure,riskEvent
produce <riskExposure, [scheduleRisk, activity$_A$, Value],0>
<riskEvent$_B$,[Risk, scheduleRisk, activity$_B$],T>
<riskEvent$_C$,[Risk, scheduleRisk, activity$_C$],T>
with newRiskExposure(currentValue, Risk, NewValue)

The **from** and **cons** parts of this rule have already been discussed; the interesting clause here is stated in the **produce** part. As in the previous case, the schedule risk `Value` for `activity`$_A$ is incremented but two further actions take place. The first one, after a delay T, alerts the `activity`$_B$ putting into the local repository named `riskEvent`$_B$ a schedule risk for the `activity`$_B$. The same action is done with respect to `activity`$_C$. When a risk becomes real for `activity`$_A$, the rule "notify" the dependent `activity`$_B$ and `activity`$_C$. It is worthwile to observe that `activity`$_B$ and `activity`$_C$ have their own RSF rules able to automatically handle the new situation deriving from the propagation of a risk involving `activity`$_A$; in other words, a greater risk for `activity`$_A$ has consequences for both `activity`$_B$ and `activity`$_C$.

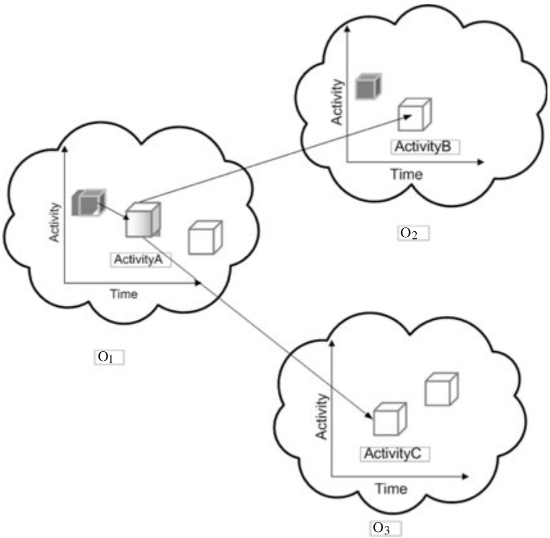


Fig. 6 Dependent activities in a VE

As soon as the risk ontology is built together with the risk matrix model containing the risk values associated to activities, the leader organization starts life cycles for each risk with a potential impact on the VE; each enterprise starts a risk life cycle too and executes the following steps according to SEI paradigm:

1. **Identify**: risk type are classified by means of a shared ontology;
2. **Analyze**: risk weight upon its own activities are evaluated by means of the risk matrix;
3. **Plan**: a mitigation action is planned or a cooperation is started with other organizations in order to learn more;
4. **Track**: each organization keeps track on the local risks whilst leader organization keeps track on the global risks that affect the entire project;
5. **Control**: each organization decides to handle the risk locally or to perform an escalation action. The leader organization has a global overview of all identified risk types and establishes if there exists correlation between two or more risks;
6. **Communicate**: the communication is guaranteed through the sending and receiving of messages (e-mail, etc.)

5 Case Study 2: The Monitoring of Environmental Risks

In this section we present a second instance of the general model applied to a distributed sensor network where the surveillance of given environmental variables is required to avoid or mitigate risks [6]. Our reference is a project for the supervising of environmental risks in specific areas called *cluster* in the territory of "Regione Campania" in the south of Italy (Fig. 7). The project, funded by POR Campania

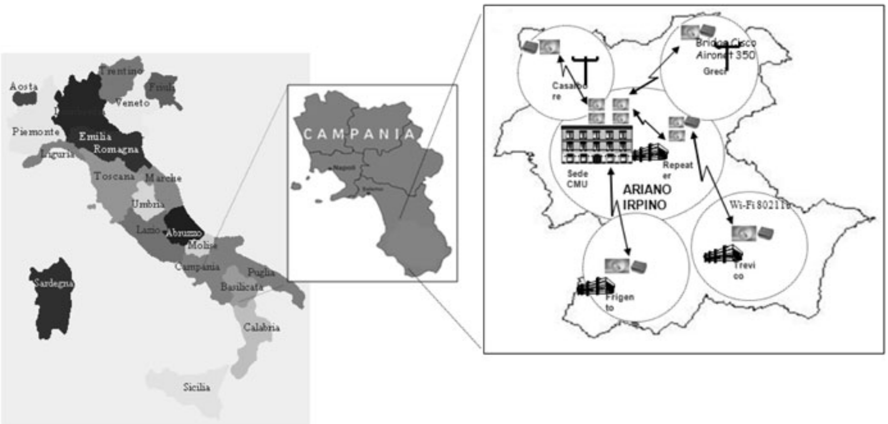


Fig. 7 The five cluster of the project

2000–2006 Misura 6.2 "Società dell'Informazione", has been realized for five clusters in the restricted area named "Comunità Montana degli Alburni". For each cluster, some hw/sw *stations* concerning the video sensor network have been installed including day&nigth and/or infrared cameras together with sensors for meteo survey, electromagnetism, earthquake, air pollution, etc.

If we consider the general metamodel, in this instance, each cluster is a concrete context that can be represented by a Local Monitor node LM_i collecting data from physical sensors that obtain information from the environment; one cluster of the network plays the role of supervisor and can be considered as the Global Monitoring Node.

In the referred project, the cluster of Ariano Irpino is the supervisor (for its central geographical and administrative position) and assumes the role of Global Monitoring node GM; indeed, in it operates a local governance with management responsibilities about the monitoring of environmental risks of the connected clusters. Following the tailoring phase of the metamodel, we first structure a specialized risk ontology as shown in Fig. 8. Each cluster has its own responsibility for risk management; it has a certain number of stations that are equipped with different sensor types (meteo multisensor: Unit WS2000, Seismograph, air pollution:Unit ETL2000, etc) in order to collect the environmental variables (methane, sulphuric acid, ammonia, ozone, temperature, wind direction, pressure, dissolved oxygen, etc).

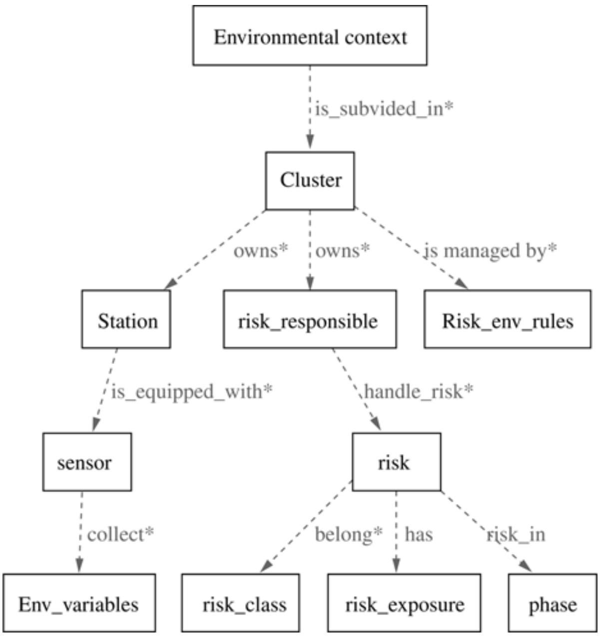


Fig. 8 A risk ontology for the environmental risk management

LM_i are placed in a particular area of the territory to put under control various types of risk. A preliminary risk analysis phase could establish that the risk of fire increases when the temperature gets over a certain threshold (e.g. 45 °C) while humidity decreases. In this way, local monitors evaluate at distinct time instants the information retrieved from the sensor network, estimating in real time the risk level and making a mitigation action when a fire risk probability increases (for example sending an alarm to local authorities).

Each Local Monitoring node is devoted to control the assigned risk types chosen among those registered during the identification phase managed by the Global Monitor node and decides the correct and more efficient action to perform when risks arise. When a critical situation happens, the Local Monitor node must evaluate the current state and take a decision in order to resolve the problem or mitigate the risk. If not enough information are available to decide autonomously, it could require other information from neighboring node. The behaviour of Local and Global Monitor are defined by means of RSF rules. Rule *R3* tries to prevent the fire extension in a wider area when the fire risk for a node becomes very high and a fast wind is blowing. The rule sends an alert to neighbouring nodes in order to activate local controls in other LM's.

R3:when the fireRisk is extreme, the rule sends an alert to selected neighbouring clusters
from <windDirection, v_1, T_1 ><windSpeed,v_2,T_1 > <riskExposure,[fireRisk, v_3],T_1 >
with v_2 = "fast", v_3 = "extreme"
produce <alert, [nodeList, "fireRisk"], 0> **with** nodeList= neighbours(LM, v_1, v_2)

where the function *neighbours* takes into account the wind direction and the wind speed to compute the list of LM nodes to alert.

When the risk exposure for fire risk becomes `extreme`, the local node may request information from its neighbours in order to evaluate the potential impact of the fire risk. In this case, it waits a response from other contacted local node and, at the same time, it alerts the State Forestry Corps for controlling the fire situation in the area.

R4: LM_i requires the fireRisk for its neighbours LM_j e LM_k
from <riskExposure,[fireRisk, v],T> **with** v= "extreme"
produce <alert, [stateForestryCorps, ["fireRisk", LM_i], 0>
<request, [LM_j, "fireRisk"], 0> <wait, [LM_j, "fireRisk", v], T
<request, [LM_k, "fireRisk"], 0> <wait, [LM_k, "fireRisk", v], T

When an alerted cluster sends back a response and its fire risk is `high`, the case of a shared risk happens. So, an alert is sent to the GM that must execute a higher level control action.

R5 escalation on shared risk
from <wait, [LM, "fireRisk", v], T_1 ><receive, [LM, "fireRisk", v1], T_2 >
with T2 <= T1, v1= "high"
cons wait
produce <alert, [GM, ["fireRisk", [LM_i,LM_j]], 0>

Once the specialized ontology equipped with risk specification rules have been defined, the risk lifecycle of each Local Monitoring Node and of Global Monitoring Node performs the following activities:

1. **Identify**: establishes the risk list and distributes their responsabilty to each Local Monitoring Node;
2. **Analyze**: each Local Monitoring Node evaluates the inputs from the sensors and historical data exploiting the RSF rules to eventually suggest a mitigation action;
3. **Plan**: a Local Monitoring Node uses data analysis to decide the better strategy to follow (e.g. plan a mitigation or a contingency action);
4. **Track**: risk status data are collected and registered in the historical data repository;
5. **Control**: each Local Monitoring Node decides to handle the risk locally or to perform an escalation action;
6. **Communicate**: the communication is guaranteed through the sending and receiving of messages.

6 Conclusions

The metamodel proposed in this paper arises from an approach to risk management based on the definition of risk ontologies. Representing the fundamental concepts of risk management and specifying the common behavior of risk management systems allows to obtain several advantages. First of all, existing conceptual analogies between risk management systems can be exploited when a new risk management system has to be implemented. Indeed, many risk concepts and rules for risk handling are similar even if the application contexts are different [3]. The metamodel represents in an abstract setting a sensor network together with transition rule patterns that can be used to specify the reactive behavior of a risk management system. Another benefit of our metamodel is the possibility of reuse. In fact, when the metamodel must be applied to a new application context, a tailoring phase uses the metamodel as a guideline for the design, with a reasonable effort, of a new risk management system. Finally, the organizations can receive the support of ICT systems to identify what risk management information is needed and what operations or behaviors are related to risk management and must be performed in order to avoid or mitigate risks. This can bring a deeper understanding of distributed risk, the check of the domain and knowledge sharing.

Due to the great number of application context and to their variety and complexity, we are aware that the metamodel do not catch all the possible aspects concerning the risk management. In some case the tailoring phase could be cumbersome and the benefit of the metamodel tailoring, that is the use of patterns and analogies, would be restricted only to a marginal part of the whole system to implement. Nonetheless, the metamodel has been successfully validated on the class of reactive systems that is the class of systems that activate themselves in response events incoming from the circumscribing environment.

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